Steady Flow of Power Law Fluids over a Pair of Cylinders in Tandem Arrangement

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The steady flow of incompressible power-law fluids over a pair of cylinders in tandem arrangement has been studied numerically. The field equations have been solved using a finite volume method based solver (FLUENT 6.2). In particular, the effects of the power-law index (0.4 ≤ n ≤ 1.8), Reynolds number (1 ≤ Re ≤ 40), and the gap ratio between the two cylinders (2 ≤ G ≤ 10) on the local and global flow characteristics such as streamline profiles, center line velocity, surface pressure coefficient, and individual and total drag coefficients, etc. have been studied in detail. The wake interference in conjunction with the power-law rheology exerts a strong influence on the flow dynamics at high Reynolds numbers, even in the steady-flow regime, whereas at low Reynolds numbers, the flow is influenced by the rheological behavior of the fluid. The increasing degree of shear-thinning behavior delays the flow separation, whereas early separation is seen in Newtonian and in shear-thickening fluids. The pressure coefficient distribution on the surface of the cylinders shows an intricate dependence on the power-law index, Reynolds number, and gap ratio. The shear-thinning behavior exerts stronger effects on the drag characteristics than those seen in Newtonian and shear-thickening fluids. Both upstream and downstream cylinders show smaller values of the individual and total drag coefficients than those for a single circular cylinder under otherwise identical conditions.

1. Introduction

Over the years, considerable research efforts have been devoted to the study of the cross-flow of Newtonian and non-Newtonian fluids over cylinders of circular and non-circular cross-sections. Indeed, depending upon the value of the Reynolds number, this flow displays a wide variety of flow regimes over a relatively small range of Reynolds numbers, even in Newtonian fluids.1,2 The fluid-flow interactions for multiple bluff bodies are very important as well as complex, yet only limited information is available in the literature, even for Newtonian fluids.1,3-4 The hydrodynamic forces and flow configurations are major criteria for the design of structures in many engineering applications, that is, tubes in tubular and in pin-type heat exchangers, which are used extensively in the cooling of electronic-components and in food and polymer processing applications. Further applications are found in the resin transfer molding process of manufacturing fiber reinforced composites, in filtration screens and aerosol filters, etc. Also, extra bodies are often used in flow-control strategies to modify, or to probe, the original wake. A pair of two cylinders in various arrangements is the simplest case of multiple bluff bodies. A thorough understanding of the flow dynamics of such an idealized case is germane to understanding the flow around complex arrangements, such as in the shell of tubular heat exchangers, in pin-type heat exchangers, and in tubular membrane modules, etc. Evidently, an improved understanding of the hydrodynamics of the flow (such as the dead-zones or the zones of maximum shear, wake size, etc.) will lead to improved design methods and ensure homogeneous product quality for temperature-sensitive materials (such as foodstuffs). In such cluster configurations, the types of interference encountered are varied such as the proximity interference, wake interference, and a combination of both. Among many possible arrangements in which two circular cylinders can be positioned relative to the cross-flow, tandem arrangement has been studied extensively. This configuration is suited to study wake interference, wherein the wake of the upstream cylinder may extend up to the downstream cylinder.4 This work is concerned with the flow of power-law fluids over a pair of cylinders in tandem arrangement.

It is probably reasonable to say that a voluminous body of information is now available on the various aspects of flow phenomena associated with the transverse flow of Newtonian fluids over a single cylinder (e.g., refs 1-7). On the other hand, many substances of multiphase nature and/or of high molecular weight encountered in industrial practices (pulp and paper suspensions, food, polymer melts and solutions, etc.) display shear-thinning and/or shear-thickening behavior.6 Owing to their high viscosity levels, these materials are generally processed in the laminar flow conditions. Therefore, it seems reasonable to begin with the analysis of purely viscous power-law type fluids, and the level of complexity can be built up gradually to accommodate other non-Newtonian characteristics such as yield stress, visco-elasticity, etc. As far as known to us, there has been no prior study on the steady cross-flow of incompressible power-law liquids over a pair of cylinders in tandem arrangement. This constitutes the main objective of this work. At the outset, it is desirable, however, to briefly recount the available limited work on the flow of Newtonian fluids over a pair of cylinders and of power-law fluids past a single cylinder to facilitate the subsequent presentation of the new results for the pair of cylinders in tandem configuration.

2. Previous Work

A pair of cylinders can geometrically be arranged in side-by-side, or tandem (or aligned), or staggered configurations with respect to the direction of the approaching flow. It is useful to note here that both types of arrangements are encountered on the shell-side of tubular heat exchangers and membrane separa-
tion units. Over the years, many previous investigations have already revealed complex flow behavior, different flow patterns, and wake interferences depending upon the relative positioning of the two cylinders, even in Newtonian fluids. A detailed classification of the different types of flow interference regimes has been proposed in the literature (e.g., refs 4, 9–13, etc.). For instance, Igarashi10,11 identified six different possible types of interferences for a pair of cylinders in tandem arrangement. Some of these features have been studied at high Reynolds numbers among others by Okajima,14 Arie et al.,15 and Xu and Zhou16 and have been recently summarized.17

Among the numerical studies on the flow around two cylinders, Stansby and Slauoti18 used the inviscid discrete-vortex method to investigate the flow around two side-by-side circular cylinders at high Reynolds numbers. Their results are in line with the available experimental studies. Subsequently, they19 used a random vortex method to model the 2D flow around two (tandem and side-by-side) circular cylinders at Re = 200. Similar flow configurations have been studied by Mittal et al.20 by using a stabilized finite element method at Re = 100 and 1000. In a recent bi-dimensional numerical analysis (using both linear stability analysis and DNS), Mizushina and Suehiro21 investigated the 2D wake instability of two tandem circular cylinders for the gap ratio (G) ranging from 2 to 7 and the Reynolds number in the range of 60 to 100. They reported the bifurcation diagram to become complex in the vicinity of the instability threshold. Subsequently, Huang et al.22 simulated the 2D unsteady flow of water in aligned and staggered arrays of cylinders at Re = 150 using FLUENT. Recently, using the mesh-free least-square-based finite difference (MLSPFD) method, Ding et al.23 have simulated the flow around two cylinders in side-by-side and tandem arrangements for two values of the Reynolds number Re = 100 and 200. They characterized the flow field by presenting the instantaneous streamlines and vorticity contours and in terms of global engineering parameters such as the Strouhal number, the mean values, and the amplitudes of drag and lift coefficients. These results are also consistent with the previous experimental findings24 in the various flow regimes and with interactive vortex shedding. More recently, Juncu24 has numerically solved the transformed vorticity-stream function formulation of the Navier–Stokes equations using a finite difference method for the steady Newtonian flow around two cylinders (of different diameters, D1 and D2) in tandem arrangement (gap ratio, G∗ = 2L2/D1, where L1 and L2 are the distances of the upstream and downstream cylinders from the origin of the coordinate systems). He presented extensive results showing the effects of the Reynolds number (based on D1), Re = 1 to 30, of the diameter ratio, D2/D1 = 0.5, 1, and 2 on the pressure coefficient, vorticity, and the individual and total drag coefficients for the two-cylinder system.

Among the experimental studies, the evolution of the flow interference behind the two side-by-side cylinders was investigated using flow-visualization methods by Barman and Wadcock.25 Similarly, Williamson26 experimentally studied this problem in the Reynolds number range of 50 to 200. He observed synchronized behavior, that is, anti-phase and in-phase features of the wakes for certain values of the gap ratio. Recently, Tasaka et al.27 have confirmed experimentally the existence of two (slow and fast) modes of vortex shedding for two circular cylinders in tandem. Their results confirm the 2D numerical predictions23 that resist finite size effects or eventual 3D instabilities. For specific gap ratios, they detected the subcritical and saddle node bifurcations that lead to hysteretic exchanges between the two modes of vortex shedding. As a result of the finite length of the cylinders, they observed slightly different ranges and transition values of the Reynolds number and gap ratios from those based on the numerical results for infinitely long cylinders (e.g., ref 21, etc.).

In contrast, the corresponding limited information for the flow of power-law fluids over a single cylinder has been summarized in recent studies.28–42 The limits of the cessation of the creeping flow regime and of the transition from the 2D steady symmetric flow to the asymmetric flow regimes have been delineated only recently for power-law fluids.28 This study shown that shear-thickening fluid behavior can advance the formation of asymmetric wakes to lower values of the Reynolds number than that in Newtonian liquids. All in all, reliable results are now available for the flow of power-law fluids over a cylinder in the 2D steady symmetric flow regime embracing the range of conditions as: Re ≤ 40, 0.2 ≤ n ≤ 2. Similarly, some information46,47 is also available for the flow past a cylinder confined in a planar channel. Aside from these results based on the solution of the complete governing equations, there have been some studies based on the boundary layer flow approximation (e.g., refs 40 and 42 and references therein).

In summary, as far as known to us, there has been no prior study dealing with the steady flow of power-law fluids over two tandem cylinders in an unconfined and cross-flow configuration. This work is concerned with the 2D steady and unconfined cross-flow of an incompressible power-law fluid over two tandem cylinders over the following ranges of conditions: Reynolds number (1 ≤ Re ≤ 40), power-law index (0.4 ≤ n ≤ 1.8), and gap ratio (2 ≤ G ≤ 10).

3. Problem Statement and Governing Equations

Consider the 2D, steady, cross-flow of an incompressible power-law liquid streaming with a uniform velocity (U0) over a pair of infinitely long circular cylinders (of equal diameter, D) in tandem arrangement (gap ratio, G = LD, where L is the center-to-center distance), as shown in part a of Figure 1. The unconfined flow condition is simulated here by enclosing the
two circular cylinders in a circular outer boundary (of diameter $D_o$), as shown schematically in part b of Figure 1. The diameter of the outer circular boundary $D_o$ is taken to be sufficiently large to minimize the boundary effects.

The continuity and momentum equations for this flow in their compact forms are written as follows,

- **Continuity equation:** $\nabla \cdot U = 0$  

- **Momentum equation:** $\rho U \nabla U + f - \nabla \sigma = 0$  

where $\rho$, $U$, $f$, and $\sigma$ are the density, velocity, body force, and the stress tensor, respectively. The stress tensor, sum of the isotropic pressure ($p$), and the deviatoric stress tensor ($\tau$), is given by

$$\sigma = -pI + \tau$$

For incompressible fluids, the extra stress tensor is given by

$$\tau = 2\eta \varepsilon(U)$$

where $\varepsilon(U)$, the components of the rate of strain tensor, are given by

$$\varepsilon(U) = \frac{(\nabla U) + (\nabla U)^T}{2}$$

For a power-law fluid, the viscosity ($\eta$) is given by

$$\eta = m(2I_2)^{n-1}$$

where $m$ is the power-law consistency index and $n$ is the power-law index of the fluid ($n < 1$, shear-thinning; $n = 1$, Newtonian; and $n > 1$, shear-thickening) and $I_2$ is the second invariant of the rate of strain tensor ($\varepsilon$). The components of the rate of the strain tensor are related to the velocity components and their derivatives and are available in standard text books (e.g., ref 43).

The physically realistic boundary conditions for this flow configuration may be written as follows:

- **At the inlet boundary:** The uniform flow condition is imposed at the inlet.

$$U_x = U_o \quad \text{and} \quad U_y = 0$$

- **On the surface of the cylinders:** The standard no-slip condition is used.

$$U_x = 0 \quad \text{and} \quad U_y = 0$$

- **At the exit boundary:** The default outflow boundary condition option in FLUENT (a zero diffusion flux for all of the flow variables) was used in this work. This choice implies that the conditions of the outflow plane are extrapolated from within the domain and as such have negligible influence on the upstream flow conditions. The extrapolation procedure used by FLUENT updates the outflow velocity and the pressure in a manner that is consistent with the fully developed flow assumption, when there is no area change at the outflow boundary. However, the gradients in the cross-stream direction may still exist at the outflow boundary. Also, the use of this condition obviates the need to prescribe a boundary condition for pressure. This is similar to the homogeneous Neumann condition, that is,

$$\frac{\partial U_x}{\partial x} = 0 \quad \text{and} \quad \frac{\partial U_y}{\partial x} = 0$$

The numerical computations have been carried out in the full computational domain, that is, without assuming midplane symmetry (Figure 1). The numerical solution of the governing equations (eqs 1–2) in conjunction with the above-noted boundary conditions (eqs 7–9) maps the flow domain in terms of the primitive variables, that is, velocity ($U_x$ and $U_y$) and pressure ($p$) fields. These, in turn, are used to deduce the local and global momentum characteristics as outlined below and detailed elsewhere.\(^\text{34,36–40}\) However, at this stage, it is useful to introduce some dimensionless parameters.

- The Reynolds number ($Re$) for power-law fluids is defined as follows

$$Re = \frac{\rho D^2 U_o^2}{m}$$

- The surface pressure coefficient ($C_p$) is defined as follows

$$C_p = \frac{\text{Static pressure}}{\text{Dynamic pressure}} = \frac{p(\theta) - p_o}{(1/2)\rho U_o^2}$$

where $p(\theta)$ is the surface pressure at an angle $\theta$ and $p_o$ is the free stream pressure at the exit boundary.

- The total drag coefficient ($C_D$), the sum of the friction and pressure components, is defined as

$$C_D = \frac{F_D}{(1/2)\rho U_o^2 D} = C_{DP} + C_{DF}$$

where $F_D$ is the drag force on the cylinder per unit length. The individual drag coefficients, $C_{DP}$ and $C_{DF}$, are calculated using the following definitions:

$$C_{DP} = \frac{F_{DP}}{(1/2)\rho U_o^2 D} = \int_S C_p n_x dS$$

where $F_{DP}$ is the pressure component of the drag force and $S$ is the surface area.

$$C_{DF} = \frac{F_{DF}}{(1/2)\rho U_o^2 D} = \frac{2^{n+1}}{Re} \int_S (\tau n_x) dS = \frac{2^{n+1}}{Re} \int_S (\tau_{xx} n_x + \tau_{xy} n_y) dS$$

where $F_{DF}$ is the frictional component of the drag force and $\textbf{n}$ (unit vector normal to the surface of the cylinder) is given as

$$\textbf{n} = \frac{x\textbf{e}_x + y\textbf{e}_y}{\sqrt{x^2 + y^2}} = n_x \textbf{e}_x + n_y \textbf{e}_y$$

where $\textbf{e}_x$ and $\textbf{e}_y$ are the $x$ and $y$ components of the unit vector, respectively, and $\tau$, the dimensionless shear stress, is expressed as

$$\tau_{ij} = n \left( \frac{\partial U_i}{\partial j} + \frac{\partial U_j}{\partial i} \right) = \left( \frac{I_2}{2} \right)^{(n-1)/2} \left( \frac{\partial U_i}{\partial j} + \frac{\partial U_j}{\partial i} \right)$$

where $\eta$ and $I_2$ are the dimensionless viscosity and second invariant of the rate of strain tensor, respectively. In the above equations, the radius of the cylinder ($D/2$) and the uniform inlet velocity ($U_o$) are used as the characteristic length and characteristic velocity, respectively.
with the algebraic multi-grid (AMG) method solver. The use of the AMG scheme can greatly reduce the number of iterations and thus the CPU time required to obtain a converged solution, particularly when the model contains a large number of control volumes. Relative convergence criteria of $10^{-10}$ for the continuity and $x$ and $y$ components of the velocity were prescribed in this work.

5. Choice of Computational Domain and Grid Size

Needless to say, the reliability and accuracy of the numerical results are contingent upon a prudent choice of the numerical parameters, namely, the optimal domain and grid sizes. In this study, the domain is characterized by the diameter ($D_o$) of the faraway cylindrical envelope of the fluid. An excessively large value of $D_o$ will warrant enormous computational resources and a small value will unduly influence the results, and hence a judicious choice of $D_o$ is vital to the accuracy of the results. Similarly, an optimal grid size should meet two conflicting requirements, namely, it should be fine enough to capture the flow field yet it should not be excessively resources intensive. The effects of these parameters ($D_o$ and grid size) on the drag coefficient values for the power-law fluid flow past a single cylinder have been explored extensively recently,

Several values of ($D_o/D$) ranging from 300 to 1300 have been used in this study to examine the role of domain size on the present numerical results. Table 1 shows the effect of domain size ($D_o/D$) on the individual and total drag coefficient values for $G = 2$, for three values of the power-law index ($n = 0.4$, 1, and 1.8) and for the extreme values of the Reynolds number ($Re = 1$, 30 and/or 40) considered in this work. The domain independence study has been carried out with grid $M_1$ detailed in Table 2. It shows a very small change ($<0.05\%$) in drag values with an increase in domain size from 1100 to 1200 and from 1200 to 1300 at Reynolds number $Re = 1$. Furthermore, the change in domain size from 300 to 400 and 400 to 500 for $Re = 40$ ($n = 0.4$ and 1) and for $Re = 30$ ($n = 1.8$) is found to a yield very small change in the drag values ($<0.18\%$). It needs to be emphasized here that the extremely small changes seen in the values of the drag coefficients are accompanied by a 2- to 3-fold increase in CPU time in the extreme condition.

Table 2. Grid Specifications (a) Details of Grids Used in Grid Independence Study, (b) Number of Grid Cells Used in Final Computations for Different Values of Gap Ratio ($G$) with the $M_4$ Grid

(a) number of cells in the computational domain ($N_{\text{cells}}$)

<table>
<thead>
<tr>
<th>grid</th>
<th>$D_o/D$</th>
<th>$N_{\text{cells}}$ for $D_o/D = 400$</th>
<th>$N_{\text{cells}}$ for $D_o/D = 1200$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>0.010</td>
<td>175 852</td>
<td>160 202</td>
</tr>
<tr>
<td>$M_2$</td>
<td>0.010</td>
<td>240 208</td>
<td>192 708</td>
</tr>
<tr>
<td>$M_3$</td>
<td>0.005</td>
<td>279 990</td>
<td>221 302</td>
</tr>
<tr>
<td>$M_4$</td>
<td>0.003</td>
<td>302 388</td>
<td>290 862</td>
</tr>
<tr>
<td>$M_5$</td>
<td>0.003</td>
<td>318 388</td>
<td>268 702</td>
</tr>
</tbody>
</table>

(b) $G$ number of cells for $D_o/D = 400$ number of cells for $D_o/D = 1200$

<table>
<thead>
<tr>
<th>gap ratio, $G$</th>
<th>$N_{\text{cells}}$ for $D_o/D = 400$</th>
<th>$N_{\text{cells}}$ for $D_o/D = 1200$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>175 852</td>
<td>160 202</td>
</tr>
<tr>
<td>3</td>
<td>240 208</td>
<td>192 708</td>
</tr>
<tr>
<td>4</td>
<td>279 990</td>
<td>221 302</td>
</tr>
<tr>
<td>6</td>
<td>302 388</td>
<td>290 862</td>
</tr>
<tr>
<td>10</td>
<td>318 388</td>
<td>268 702</td>
</tr>
<tr>
<td>$\infty$</td>
<td>245 050</td>
<td>309 050</td>
</tr>
</tbody>
</table>

$\ast$ $D_o/D$: grid spacing in the proximity of the cylinders; $\ast$ $D_o/D = 200$. $N$: number of grid points on the surface of the cylinder.
Table 4. Comparison of the Newtonian (n = 1) Flow Results for a Single Cylinder (G = ∞) and the Two Cylinders in Tandem Arrangement. (CDo: Average Drag Coefficient; ClD: Total Lift Coefficient; Sr: Strouhal Number)

<table>
<thead>
<tr>
<th>Re</th>
<th>Source</th>
<th>Upstream Cylinder</th>
<th>Downstream Cylinder</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>CDo,1 ±C1,1</td>
<td>St1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>Mittal et al.20</td>
<td>1.275</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>Ding et al.21</td>
<td>1.163</td>
<td>0.0</td>
</tr>
<tr>
<td>100</td>
<td>Mittal et al.20</td>
<td>1.446 ±0.015</td>
<td>0.413</td>
</tr>
<tr>
<td></td>
<td>Ding et al.22</td>
<td>1.433 ±0.015</td>
<td>0.403</td>
</tr>
<tr>
<td>1</td>
<td>Present Results</td>
<td>1.329 ±0.013</td>
<td>0.330</td>
</tr>
<tr>
<td>Juncu24</td>
<td>3.8272</td>
<td>3.9621</td>
<td>7.7893</td>
</tr>
<tr>
<td>2</td>
<td>Present Results</td>
<td>3.9150</td>
<td>3.9900</td>
</tr>
<tr>
<td>Juncu24</td>
<td>2.6880</td>
<td>2.6677</td>
<td>5.3557</td>
</tr>
<tr>
<td>5</td>
<td>Present Results</td>
<td>2.7510</td>
<td>2.6820</td>
</tr>
<tr>
<td>Juncu24</td>
<td>1.7785</td>
<td>1.6074</td>
<td>3.3859</td>
</tr>
<tr>
<td>10</td>
<td>Present Results</td>
<td>1.8190</td>
<td>1.6170</td>
</tr>
<tr>
<td>Juncu24</td>
<td>1.3890</td>
<td>1.1040</td>
<td>2.4711</td>
</tr>
<tr>
<td>20</td>
<td>Present Results</td>
<td>1.1046</td>
<td>0.7485</td>
</tr>
<tr>
<td>Juncu24</td>
<td>1.1350</td>
<td>0.7500</td>
<td>1.8850</td>
</tr>
</tbody>
</table>

1200 are believed to be adequate in the Reynolds number range of 1 ≤ Re ≤ 5 and 5 ≤ Re ≤ 40, respectively, for gap ratio 2 ≤ G ≤ 10 over the power-law index range (0.4 ≤ n ≤ 1.8) considered here, to obtain the results that are essentially free from domain effects.

Having fixed the domain size, the grid independence study has been carried out for five non-uniform unstructured grids (M1, M2, M3, M4, and M5) for the extreme values of the Reynolds number (Re = 1, 30 and/or 40), for three values of the power-law index (n = 0.4, 1, and 1.8) and for the gap ratio G = 2. The grid details are shown in Table 3. The influence of the grid size on the individual and total drag coefficients for the two-cylinder case is shown in Table 3. It can be clearly seen from these results that, in moving from grid M1 to M5, the drag coefficient values show a very small change, the maximum change being 0.45, 0.17, and 0.11% at Re = 1 for n = 0.4, 1, and 1.8, respectively. The corresponding changes at Re = 40 are seen to be 0.62 and 0.6% for n = 0.4 and 1, respectively, 2.48% at Re = 30 and n = 1.8. In view of these negligible changes (accompanied by an enormous increase in the computational time), the grid M4 is believed to be sufficiently refined to resolve the momentum transfer phenomena with acceptable levels of accuracy within the range of conditions of interest here.

The number of grid cells with grid M4 for different values of the gap ratio (G) is also shown in Table 2. Finally, to add further weight to our claim for the accuracy of the results, the numerical results obtained herein have been compared with the literature values in the next section in the limiting case of Newtonian fluid behavior.

6. Results and Discussion

In this work, the 2D steady-flow computations have been carried out for the following values of the dimensionless parameters: Reynolds number, Re = 1, 2, 5, 10, 20, and 30 and/or 40; the power-law index, n = 0.4, 0.6, 1, 1.4, and 1.8, thereby covering both shear-thinning (n < 1) and shear-thickening (n > 1) fluids, and for five values of the gap ratio, G = 2, 3, 4, 6, and 10. Because the flow of power-law fluids over a single cylinder is known38 to become asymmetric for Re = 40 at n = 1.8, the new results for this power-law index are restricted to Re ≤ 30. Without assuming the flow symmetry
about the midplane, the results have been obtained using the full computational domain (Figure 1). However, prior to presenting the new results, it is appropriate to validate the solution procedure to ascertain the accuracy and reliability of the results presented herein.

6.1. Validation of Results. Because extensive validation for the case of a single cylinder is dealt with in detail elsewhere, the present results for the flow of Newtonian fluids over two cylinders in tandem arrangement are compared in Table 4 with the available literature values. Because only limited results are available for the flow over the two-cylinder system in the range of conditions studied herein, limited time-dependent computations have also been carried out at Re = 100 for the purpose of validation only (Table 4). An excellent correspondence can be seen in this table. The orders of the deviations seen in Table 4 are not at all uncommon in such numerical studies due to the inherent differences stemming from different flow schematics, grid sizes, and solution methodologies, etc. For instance, the rectangular domain used by Mittal et al. employs \( L_u = 5D, \quad L_d = 16D \) and \( H = 16D \), where \( L_u, \quad L_d, \quad \) and \( H \) are the upstream and downstream lengths and height of the computational domain, respectively. The present results for this case (shown in Table 4) are also based on this domain size. On the other hand, Ding et al. also used a rectangular domain, but with \( L_d = 24D \) and \( H = 25D \), respectively. Juncu has used the orthogonal curvilinear transformed stream function–vorticity formulation to solve this problem. Notwithstanding these inherent differences in these studies (including the use of finite element, finite difference, and MLSFD methods), the correspondence seen in Table 4 is regarded to be satisfactory and acceptable.

On the basis of these comparisons together with our previous experience and coupled with the fact that the numerical predictions for power-law fluids tend to be less accurate, the present results for a pair of cylinders are believed to be reliable to within \( \pm 2-3\% \).
6.2. Detailed Flow Kinematics. Some physical insights into the nature of the flow can be gained by examining the streamline patterns, center line velocity, and surface pressure profiles.

6.2.1. Streamline Profiles. Representative plots showing the dependence of the streamline patterns in the vicinity of the cylinders on the Reynolds number ($Re$), power-law index ($n$), and gap ratio ($G$) are presented in Figures 2 and 3. For fixed values of the power-law index ($n$) and gap ratio ($G$), the size of the recirculation zone grows and the point of the separation moves forward on the surface of the cylinders for both shear-thinning ($n < 1$) and shear-thickening ($n > 1$) fluids as the Reynolds number ($Re$) is progressively increased. For fixed values of the Reynolds number ($Re$) and gap ratio ($G$), fluids at low values of the gap ratio ($G$) changes from Newtonian ($n = 1$) to shear-thickening ($n > 1$), the recirculation region grows in size. The shear-thinning ($n < 1$) fluid behavior shows complex dependence of the wake size for all values of $Re$ and $G$. For small values of the Reynolds number ($Re \leq 5$), no separation was observed for any value of the power-law index ($n$) and gap ratio ($G$). Similarly, irrespective of the value of the gap ratio, no flow separation occurred in highly shear-thinning fluids ($n < 0.4$) for Reynolds numbers up to 20. In shear-thickening ($n > 1$) fluids, at low values of the gap ratio ($G \leq 4$), the wake interference phenomena can clearly be seen in Figures 2 and 3, where the wake of the upstream cylinder is being suppressed due to the downstream cylinder being too close to it. For shear-thinning fluids at low Reynolds numbers, the viscosity becomes very large as the shear rate decreases, and hence it tends to infinity, far away from the cylinder when shear rate is zero. On the other hand, for shear-thickening fluids ($n > 1$), the inertial

Figure 4. Velocity profiles at the horizontal center line ($y = 0$) for different values of the power-law index ($n$) at a gap ratio of $G = 2$ for (a) $Re = 1$, (b) $Re = 10$, and (c) $Re = 40$. The locations $X_{R1}$, $X_{F2}$, and $X_{R2}$ are the rear stagnation point ($\theta = \pi$) of the first cylinder and the front ($\theta = 0$) and rear ($\theta = \pi$) stagnation points of the second cylinder, respectively.
effects dominate, even when far away from the cylinders. Thus, the gap ratio shows stronger dependence on the wake interference in shear-thickening fluids and/or in high Reynolds number flows. It is, however, appropriate to add here that, in Newtonian fluids, the flow separates at about \( Re \approx 5 \) for a single cylinder, but depending upon the gap between the two cylinders, the presence of the downstream cylinder can cause the flow to separate at much smaller values of the Reynolds number, as observed in experiments with a viscous glycerine solution for \( G \approx 2 \) and \( Re \approx 0.01 \). However, no separation was observed in the present study, even at \( Re = 1 \).

6.2.2. Center-Line Velocity Profile. Figures 4–6 show the variation of the dimensionless \( x \)-component of the velocity \( (U_{x} = U_{x}/U_{o}) \) along the center line \( y = 0 \) with the Reynolds number \( (Re) \), flow behavior index \( (n) \), and gap ratio \( (G) \). The left side of the figures shows the variation of the velocity along the center line connecting the two cylinders, that is, \( x_{R1} \), from the rear stagnation point \( (\theta = \pi) \) of the upstream cylinder to \( x_{F2} \), the front stagnation point \( (\theta = 0) \) of the downstream cylinder. In the right-side figures, the corresponding velocity variation from \( x_{F2} \) to the downstream center line. At a fixed location \( (x,0) \), the center line velocity increases with the increasing value of the power-law index \( (n) \) for all values of the Reynolds number \( (Re) \) and gap ratio \( (G) \). At low values of the gap ratio \( (G = 2) \), the dimensionless velocity \( (U_{x}^*) \) in the region between the two cylinders is seen to be small (and negative suggesting reverse flow) and it decreases as one moves away from the cylinders.

Figure 5. Velocity profiles at the horizontal center line \( y = 0 \) for different values of the power-law index \( (n) \) at a gap ratio of \( G = 4 \) for (a) \( Re = 1 \), (b) \( Re = 10 \), and (c) \( Re = 40 \).
The negative values imply flow separation, but the values are too small to be captured in streamline patterns. Because of such small values, the streamline patterns in Figures 2 and 3 show no streamlines in this region. The increasing value of the gap ratio ($G$) shows quite an interesting dependence of the velocity on the Reynolds number and the power-law index in both shear-thinning and shear-thickening fluids. The magnitude of the velocity is seen to be maximum at the midpoint ($G/2$ distance forward from the upstream cylinder) of the cylinders at low values of the gap ratio and low values of the Reynolds number (parts a.1–c.1 of Figure 4, part a.1 of Figure 5, and part a.1 of Figure 6). This point is seen to move forward with the increasing value of the gap ratio ($G$). The velocity ($U_x'$) profiles along the center line of the downstream cylinder show qualitatively similar dependence on the power-law index ($n$), Reynolds numbers ($Re$), and the gap ratio ($G$). The dimensionless velocity ($U_x''$) in the rear of the downstream cylinder is seen to be continuously increasing along the center line for all values of the power-law index; the increasing values of Reynolds number show a decrease (from zero to negative) followed by an increase in the values.

6.2.3. Pressure Coefficient ($C_p$) on the Surface of Cylinders. Representative results showing the variation of the pressure coefficient ($C_p$) on the surface of the upstream and downstream cylinders are plotted in Figures 7–9 for a range of values of the power-law index ($n$), for three values of the Reynolds number ($Re = 1, 10, and 40$), and for a range of values of $G$. For fixed values of the Reynolds number, the power-law index,
and the gap ratio, the variation of the pressure coefficient over the upstream cylinder shows a behavior qualitatively similar to that seen for a single cylinder.\textsuperscript{34,40} Because of varying extents of interferences, the values for the downstream cylinder show quite a complex dependence on the dimensionless parameters. The $C_P$ values are always seen to be higher for the upstream cylinder than those for the downstream cylinder, under otherwise identical conditions. For both upstream and downstream cylinders, the pressure profiles show symmetric patterns around the midplane, thereby suggesting the flow to be symmetric over the ranges of conditions studied herein.

For fixed values of the Reynolds number ($Re$), power-law index ($n$), and gap ratio ($G$), the pressure coefficient ($C_P$) for the upstream cylinder is seen to decrease from its maximum value at the front stagnation point ($\theta = 0$) along the surface toward the rear of the cylinder. In the absence of flow separation, the decrease in the pressure coefficient continues all the way up to the rear stagnation point ($\theta = \pi$), where it reaches the minimum value ($C_{P_{\text{min}}}$). On the other hand, the minimum value of the pressure coefficient ($C_{P_{\text{min}}}$) occurs at the point of separation, $\theta = \theta_s (< \pi)$, and beyond $\theta > \theta_s$, it increases up to $\theta = \pi$. For a fixed value of the flow behavior index ($n$), the

---

**Figure 7.** Variation of the pressure coefficient over the surface of the upstream (left) and downstream (right) cylinders for different values of power-law index ($n$), for a gap ratio of $G = 2$ and (a) $Re = 1$, (b) $Re = 10$, and (c) $Re = 40$. $\theta = 0$ and $\theta = 180$ correspond to the front and rear stagnation points, respectively.
point of minimum pressure coefficient ($C_{p,\text{min}}$) on the surface of the upstream cylinder shifts toward the front stagnation point ($\theta = 0$) with a gradual increase in the Reynolds number ($Re$). The pressure coefficient over the surface of both cylinders is higher for shear-thinning ($n < 1$) fluids than that for Newtonian ($n = 1$) and lower for shear-thickening ($n > 1$) fluids in the upstream side of the cylinders; the behavior is seen to switch over in the downstream side of the cylinders. At small values of the gap ratio ($G$), the values of the pressure coefficient at the front and rear stagnation points, that is, $C_p(0)$ and $C_p(\pi)$, are seen to be strongly influenced by the value of the power-law index ($n$) at low Reynolds numbers ($Re$).

6.3. Macroscopic Characteristics. In this section, the roles of the flow behavior index ($n$), the Reynolds number ($Re$), and the gap ratio ($G$) on the individual and total drag coefficients are discussed.

6.3.1. Pressure Drag Coefficient ($C_{dp}$). Table 5 shows the influence of the Reynolds number ($Re$), power-law index ($n$), and gap ratio ($G$) on the pressure drag coefficient for the upstream ($C_{dp,1}$) and downstream ($C_{dp,2}$) cylinders, respectively. Also, included in table are the corresponding values for a single cylinder ($G = \infty$). For fixed values of the Reynolds number ($Re$) and gap ratio ($G$), the pressure drag coefficient ($C_{dp}$) increases with the decreasing value of the power-law index ($n$).
for both cylinders. The shear-thinning \((n < 1)\) always yields a higher value of the pressure drag coefficient \((C_{DP})\) than the corresponding Newtonian \((n = 1)\) values; an opposite trend is seen for shear-thickening \((n > 1)\) fluids. Table 5 also shows a stronger effect of the power-law index \((n)\) on the pressure drag coefficient \((C_{DP})\) in shear-thinning \((n < 1)\) fluids than that in shear-thickening \((n > 1)\) fluids. For fixed values of \(Re, n,\) and \(G,\) the upstream cylinder always shows a higher value of the pressure drag coefficient \((C_{DP})\) than that for the downstream cylinder, that is, \(C_{DP,1} > C_{DP,2}\). The highly shear-thinning fluid \((n = 0.4)\), largest gap ratio \((G = 10)\), and small value of the Reynolds number \((Re = 1)\) show the smallest value of the ratio \((C_{DP,1}/C_{DP,2})\), which increases with the increasing value of the power-law index \((n)\) and of the Reynolds number \((Re)\). The increase in the gap ratio \((G)\) exerts an opposite influence on the pressure drag for two cylinders. The pressure drag coefficients for both cylinders are always smaller than that for a single cylinder \((C_{DP} > C_{DP,1} > C_{DP,2})\), which is clearly due to the interference between the flow fields brought about by the proximity of the two cylinders. For instance, for \(G = 2\), the ratio of the pressure drag coefficients for the two cylinders \((C_{DP,1}/C_{DP,2})\) increases from 1.18 to 4.38 as the power-law index \((n)\) was increased from 0.4 to 1.8 for a Reynolds number \((Re)\) of 1 and from 8.65 to 1323 as the power-law index \((n)\) was increased from 0.4 to 1.4 for a Reynolds number of 40, respectively. On the other hand, the change in the gap ratio \((G)\)
Table 5. Dependence of Pressure Drag Coefficient ($C_{DP}$) on the Reynolds Number ($Re$), Power-law Index ($n$), and Gap Ratio ($G$)

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<thead>
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<th>G</th>
<th>$C_{DP,1}$</th>
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</table>

from 2 to 10 decreases this ratio ($C_{DP,1}/C_{DP,2}$) from 1.18 to 1.05 and 4.38 to 2.65 at $n = 0.4$ and $n = 1.8$, respectively, for $Re = 1$. The corresponding decrease for $Re = 40$ was seen to be from 8.65 to 1.43 and 1323 to 3.59 at $n = 0.4$ and 1.4, respectively.

It can also be seen from Table 5 that the influence of the dimensionless parameters on the pressure drag coefficient is stronger for the downstream cylinder than that for the upstream cylinder. These trends clearly suggest strong wake interference on the behavior of the downstream cylinder. At small values of $G$, the pressure drag values for the two cylinders are quite close, as no wake is formed under these conditions. On the other hand, the influence of power-law index ($n$) is seen to be stronger on the pressure drag coefficient of the downstream cylinder ($C_{DP,2}$) than that for the upstream cylinder ($C_{DP,1}$) at such low Reynolds numbers. However, the effect is higher in shear-thinning ($n < 1$) fluids than that in Newtonian/shear-thickening fluids and for the downstream cylinder than for the upstream cylinder. For instance, for $G = 2$ as the Reynolds number ($Re$) is increased from 1 to 20, the $C_{DP,1}$ values decrease by factors of 8.15, 3.46, and 2.26 at $n = 0.2, 1$, and 1.8, respectively, and the corresponding changes in the values for the downstream cylinder ($C_{DP,2}$) are 25, 13.17, and 11.97, respectively. The pressure drag coefficients for the upstream cylinder show an opposite dependence on the power-law index ($n$) (i.e., decrease in $C_{DP,1}$ with decreasing values of $n$) at high values of the Reynolds numbers ($Re = 40$) and small gap ratios ($G = 2$ and 3). It should also be noted that at high values of the Reynolds number ($Re$) and the power-law index ($n$), the values of the pressure drag coefficient for the second cylinder are seen to be very close to zero, for example, $C_{DP,2} = 0.0007$ at $Re = 40$ and $n = 1.4$. It is probably due to the fact that the trailing cylinder is completely within the wake of the front cylinder.

To elucidate the role of the power-law index ($n$) on the drag characteristics, the drag coefficient values have been normalized using the corresponding Newtonian values, under otherwise identical conditions, defined as follows:

$$
\phi^n = \frac{\phi(n)}{\phi(n = 1)} \quad \text{where,} \quad \phi = C_{DP,1}, C_{DF,1}, C_{D,1}, C_{DP,2}, C_{DF,2}, C_{D,2}
$$

Figure 10 shows the normalized pressure drag coefficients, and both $C_{DP,1}^n$ and $C_{DP,2}^n$ are $> 1$ in shear-thinning fluids, whereas they are $< 1$ in shear-thickening fluids. As the fluid
behavior changes from the Newtonian ($n = 1$) to shear-thinning fluids ($n < 1$), the normalized values are seen to increase with the decreasing value of the power-law index ($n$) and Reynolds number ($Re$). For all values of the Reynolds number ($Re$) and gap ratio ($G$), the downstream cylinder shows a somewhat stronger influence of the flow behavior index ($n$) on the pressure drag coefficients ($C_{DP,2}^N$) than the upstream cylinder. The gap ratio ($G$) is seen to have only a weak effect on the normalized pressure drag coefficient for both cylinders. In shear-thickening fluids ($n > 1$), the normalized values ($C_{DP,1}^N$) are seen to be influenced by the power-law index ($n$) in a manner opposite to that seen in the shear-thinning ($n < 1$) fluids. Furthermore, for all values of the gap ratio ($G$), $C_{DP,2}$ is seen to be strongly influenced by the power-law rheology ($n$) but it is almost independent of the low Reynolds number ($Re$).

Similarly, to delineate the role of the gap ratio, $G$, in an unambiguous manner, it is useful to define the normalized drag coefficient as follows:

$$\phi^{SC} = \frac{C_{DP}}{C_{DF}} \frac{C_{D,2}^{\infty}}{C_{D}^{\infty}}$$

where, $\phi = C_{DP,1}C_{DF,1}C_{D,1}, C_{DP,2}C_{DF,2}C_{D,2}$

$$\phi = C_{DP}C_{DF}C_D \text{ (for single cylinder)}$$

The normalized values of the pressure drag coefficient for both cylinders are always found to be less than one (Figure 11), which implies strong interference between the two cylinders over the range of the gap ratio ($2 \leq G \leq 10$). For the upstream cylinder, the normalized values are seen to increase with the increasing value of the power-law index ($n$) at small values of the gap ratio ($G$) and the Reynolds number ($Re$); complex
dependence is seen at high $Re$ and $G$. The low values of the gap ratio ($G$) and the power-lax index ($n < 1$) exert a strong effect on the normalized values ($C_{DP}^{SC}$). This is due to the wake interference at a low gap ratio ($G$), which becomes more prominent with increasing values of the Reynolds number ($Re$). On the other hand, at low Reynolds numbers ($Re$), the close proximity of the two cylinders causes the formation of a stagnant zone in between the two cylinders, as can be seen from the streamline patterns (Figures 2 and 3). As a result of this stagnant region, the drag values for the upstream cylinder are seen to be very small compared to the single cylinder values, whereas they are quite comparable with the increasing values of the Reynolds numbers and/or in shear-thickening fluids. Increasing the value of the gap ratio ($G$) reduces the wake interference and the values for single cylinders and the upstream cylinders seem to be approached. The normalized pressure drag coefficient values for the downstream cylinder ($C_{DP,2}^{SC}$, right) show a completely opposite dependence on the dimensionless parameters to that for the upstream cylinder ($C_{DP,1}^{SC}$, left). Over the range of the conditions, the normalized values are seen to decrease with the increasing value of the power-law index ($n$), increasing value of the Reynolds number ($Re$) and the decreasing value of the gap ratio ($G$). Once again, the values are seen to be strongly dependent on the power-law index ($n$) at low values of Reynolds numbers ($Re$) and gap ratio ($G$).

6.3.2. Friction Drag Coefficient ($C_{DF}$). Table 6 shows the effects of the Reynolds number ($Re$), power-law index ($n$), and gap ratio ($G$) on the friction drag coefficient ($C_{DF}$). For the sake of completeness, the corresponding values for a single cylinder.
(G = ∞) are also included in this table. The friction drag coefficient values for both cylinders decrease with an increase in the Reynolds number (Re) and decrease in the gap ratio (G).

For fixed values of the gap ratio (G), the friction drag coefficient (C_{DF}) increases with the decreasing value of the power-law index (n) for the upstream cylinder at low Reynolds numbers (Re = 1 to 5) and for downstream cylinder over the entire range of conditions studied herein. Here also, the flow behavior index (n) exerts a stronger influence at low Reynolds numbers than at high Reynolds numbers. At high Reynolds numbers (5 ≤ Re ≤ 40), the friction drag coefficient for the upstream cylinder (C_{DF,1}) decreases with the decreasing values of n for all values of G. For the downstream cylinder, C_{DF,2} reaches a minimum value in a highly shear-thickening fluid, n = 1.4, at the highest value of the Reynolds number, Re = 40 for all the smallest value of G = 2. The friction drag coefficient values for both cylinders over the range 2 ≤ G ≤ 10 are always smaller than those for a single cylinder. As expected, the smaller the gap G, the larger the deviations from the single cylinder values. For fixed values of the dimensionless parameters (Re, n, and G), the upstream cylinder always shows a higher value of the friction drag coefficient (C_{DF}) than the downstream cylinder. The increase in the gap ratio (G) shows an opposite dependence on the friction drag for two cylinders. For instance, the ratio (C_{DF,1}/C_{DF,2}) at Re = 1 increases from 1.08 to 2.92 and from 1.04 to 2.45 as the power-law index (n) is increased from 0.4 to 1.8 for the gap ratio, G = 2 and 10, respectively. The corresponding changes for Re = 40 are seen to be from 2.01 to 6.28 and from 1.12 to 3.13 as the power-law index (n) is increased from 0.4 to 1.4.

Figure 12 shows the variation of the normalized friction drag coefficients C_{DF,1} and C_{DF,2}. Over the range of conditions, the variation of the normalized friction drag coefficient for the upstream cylinder shows a mirror image (Figure 12) in shear-thinning and shear-thickening fluids, respectively. For all values of gap ratio (G), the dependence of C_{DF,1} on n is seen to switch over at Re = 10. For Re < 10, the normalized values for the upstream cylinder (C_{DF,1}) decrease as the fluid behavior changes from highly shear-thinning (n = 0.4) to Newtonian (n = 1) and finally to highly shear-thickening (n = 1.8). At Re = 10, these values are seen to be almost independent of the power-law index (n) for all values of G. A further increase in the value of the Reynolds number (Re > 10) shows a completely reverse
dependence for the upstream cylinder ($C_{DF,1}^N$) with the increasing value of the power-law index ($n$). The normalized friction drag coefficient for the downstream cylinder ($C_{DF,2}^N$) shows a dependence on the power-law rheology ($n$), which is qualitatively similar to that seen for the normalized pressure drag coefficient in Figure 10. Here, also the downstream cylinder shows a stronger dependence of the normalized values on the power-law index ($n$) than that for the upstream cylinder.

Figure 13 depicts the dependence of the normalized friction drag coefficient for the upstream ($C_{DF,1}^N$, left) and the downstream ($C_{DF,2}^N$, right) cylinders on $Re$, $n$, and $G$. The dependences seen in this Figure are qualitatively similar to that for the normalized pressure drag coefficient as that seen in Figure 11.

6.3.3. Total Drag Coefficient ($C_D$). The dependence of the total drag coefficients, $C_D = C_{DP} + C_{DF}$, on the Reynolds number ($Re$), the power-law index ($n$), and the gap ratio ($G$) is shown in Figure 14. The total drag coefficient ($C_D$) shows a dependence on the dimensionless parameters ($Re$, $n$, $G$) similar to the friction drag coefficient ($C_{DF}$) for both upstream and downstream cylinders. The total drag coefficient ($C_D$) values
for both cylinders decrease with the decreasing value of the gap ratio \((G)\). For fixed values of the gap ratio \((G)\), the total drag coefficient \((C_D)\) increases with the decreasing value of the power-law index \((n)\) for the upstream cylinder at low Reynolds numbers \((Re \leq 1\) and \(5)) and for the downstream cylinder. At high values of the Reynolds numbers \((5 \leq Re \leq 40))\), the total drag coefficients for the upstream cylinder \((C_{D,1})\) show an opposite dependence on the power-law index \((n)\) for all values of the gap ratio \((G)\).

The influence of the power-law rheology \((n)\) on the total drag coefficient for an upstream \((C_{D,1})\) and downstream \((C_{D,2})\) cylinders for the range of dimensionless parameters is shown in Figure 15. The dependence of the normalized total drag coefficient on the power-law index \((n)\) is seen to be qualitatively similar to that seen for normalized pressure and friction drag coefficients, seen in Figures 10 and 12, respectively. The normalized total drag coefficient for the two cylinders also shows a dependence (Figure 16) on \(Re\), \(n\), and \(G\), which is qualitatively similar to that of the normalized pressure and friction drag coefficients seen in Figures 11 and 13.

Broadly, shear-thinning \((n < 1)\) fluid behavior always yields higher values of the drag coefficient \((C_D)\) than the corresponding

Figure 13. Dependence of the normalized friction drag coefficient for an upstream \((C_{DF,1})\) and for the downstream \((C_{DF,2})\) cylinders on the Reynolds number \((Re)\) and power-law index \((n)\) for gap ratios of \((a) G = 2\), \((b) G = 4\), and \((c) G = 10\).
Newtonian \((n = 1)\) values; an opposite trend is seen for the shear-thickening \((n > 1)\) fluids. These figures also show a stronger effect of power-law index \((n)\) on the total drag coefficient \((C_D)\) in shear-thinning \((n < 1)\) fluids than that in shear-thickening \((n > 1)\) fluids.

6.3.4. Relative Contributions of \(C_{DP}\) and \(C_{DF}\). To elucidate the role of the gap ratio \((G)\) on the relative contributions of the individual drag components, a drag ratio \((C_{DR} = C_{DP} / C_{DF})\) has been plotted in Figure 17 for a range of values of \(Re, n,\) and \(G\). For fixed values of the Reynolds number \((Re)\) and of the power-law index \((n)\), the drag ratio \((C_{DR})\) for both cylinders increases with the increasing value of gap ratio \((G)\), thereby suggesting no or little interaction between the two cylinders. The change in fluid behavior from Newtonian \((n = 1)\) to shear-thinning \((n = 0.4)\) enhances the relative contribution of the pressure component over the range of conditions, whereas the drag ratio \((C_{DR})\) reduces with the increasing value of the power-law index for shear-thickening fluids \((n > 1)\). In shear-thickening fluids, the drag ratio of the second (downstream) cylinder \((C_{DR,2})\) shows a very strong dependence on the power-law index \((n)\), especially at high Reynolds numbers.

At low Reynolds numbers \((Re = 1)\), the gap ratio \((G)\) is seen to have a very small effect on the drag ratio for the upstream cylinder \((C_{DR,1})\) in shear-thickening fluids \((n > 1)\). With the increasing value of the Reynolds number \((Re)\), the \(C_{DR,1}\) values are seen to be almost independent of the gap ratio \((G)\). These

Figure 14. Dependence of the total drag coefficients for an upstream \((C_D,1)\) and for the downstream \((C_D,2)\) cylinders on the power-law index \((n)\) and the gap ratio \((G)\) for Reynolds numbers of (a) \(Re = 1\), (b) \(Re = 2\), (c) \(Re = 5\), (d) \(Re = 10\), (e) \(Re = 20\), and (f) \(Re = 40\) \((0.4 \leq n \leq 1.4)\) and 30 \((n = 1.8)\).
values are, however, higher for the upstream cylinder than that for the downstream cylinder, under otherwise identical conditions. For instance, at $Re = 1$ as the gap ratio ($G$) increases from 2 to 10, $C_{DR,1}$ changes from 1.52 to 1.75, from 0.97 to 1.00, and from 0.82 to 0.80 for power-law indexes $n = 0.4$, 1, and 1.8, respectively. The corresponding changes at $Re = 40$ are almost nonexistent over the same ranges of conditions. The change in the drag ratio for the downstream cylinder ($C_{DR,2}$) with the gap ratio is rather strong, for example, the value changes from 1.39 to 1.73, from 0.83 to 0.99, and from 0.55 to 0.74 with an increase in the gap ratio ($G$) from 2 to 10 at $Re = 1$ for $n = 0.4$, 1, and 1.8, respectively. The corresponding changes at $Re = 40$ and for $n = 0.4$ and 1 are from 0.89 to 3.04 and from 0.28 to 1.56, and at $Re = 30$ and $n = 1.8$ is 0.034 to 1.04. This clearly shows that the relative contribution of the two components is strongly dependent on the proximity of the two cylinders in shear-thinning fluids ($n < 1$) for the upstream cylinder, whereas this dependence is weak in shear-thickening ($n > 1$) fluids. On the other hand, the downstream cylinder shows a stronger dependence on both the gap ratio ($G$) and the

Figure 15. Dependence of the normalized total drag coefficient for an upstream ($C_{D1,N}$) and for the downstream ($C_{D2,N}$) cylinders on the Reynolds number ($Re$) and the power-law index ($n$) for gap ratios of (a) $G = 2$, (b) $G = 4$, and (c) $G = 10$. 
flow behavior index \((n)\), which accentuates with the increasing value of the Reynolds number \((Re)\).

In summary, the flow characteristics of power-law fluids past a pair of cylinders in a tandem arrangement are seen to be influenced in an intricate manner by the value of the Reynolds number \((Re)\), the power-law index \((n)\), and the gap ratio \((G)\). At high values of the Reynolds numbers, the wake interferences are more prominent when the gap ratio \((G)\) is very small. On the other hand, when the cylinders are placed far from each other \((G \rightarrow \infty)\), no wake interference will occur and the two cylinders would behave in a non-interfering manner. This interplay is further accentuated by the fact that, even at low Reynolds numbers, the viscous terms in the momentum equations are highly nonlinear for power-law fluids. As the Reynolds number is increased, the flow is governed by two nonlinear terms, namely, inertial and viscous, which scale differently with velocity. For instance, the viscous forces approximately scale as \(\sim U_o^n\), whereas the inertial forces scale as \(\sim U_o^2\). Thus,
keeping everything else fixed, the decreasing value of the power-law index \( n \) suggests diminishing importance of the viscous effects for shear-thinning \((n < 1)\) fluids, whereas the inertial terms will still scale as \( \propto U_o^2 \). On the other hand, viscous effects are likely to grow with the increasing value of the power-law index \( n \) for a shear-thickening \((n > 1)\) fluid. For the extreme case of \( n = 1.8 \), the viscous terms will also scale as \( \sim U_o^{1.8} \), almost identical to the inertial term. These nonlinear interactions in conjunction with the distance between the two cylinders exert a strong influence on the momentum transfer characteristics. Even though when the cylinders are placed closed to each other (small values of \( G \)) there is no wake interference at low Reynolds numbers, the power-law rheology exerts a strong influence on the flow field and drag phenomena. It is believed that these different kinds of dependencies on the flow behavior index and velocity are also responsible for the non-monotonic behavior of kinematics of the flow as seen in this work.

7. Concluding Remarks

Extensive numerical results on the momentum characteristics of the steady unconfined flow of power-law fluids over a pair...
of cylinders in tandem arrangement have been studied over wide ranges of conditions as: $1 \leq Re \leq 40$, $0.4 \leq n \leq 1.8$, and for five values of the gap ratio ($G = 2, 3, 4, 6, \text{and } 10$). The effects of the dimensionless parameters ($Re$, $n$, $G$) on the detailed kinematics of the flow (streamline patterns, velocity, pressure profiles) and the drag phenomena (individual and total drag coefficients) are presented in detail. The wake interference in conjunction with the power-law rheology is seen to be a strong influence at high Reynolds numbers, whereas at low Reynolds numbers the flow is altered by the fluid behavior. The flow separation is seen to be delayed in shear-thinning fluids as compared to that in Newtonian and in shear-thickening fluids. The pressure coefficient distribution over the surface of the cylinders shows a complex dependence on the power-law index, Reynolds number, and gap ratio. The shear-thinning fluids show stronger effects on the drag characteristics than in Newtonian and in shear-thickening fluids. The drag coefficient values show a dependence on the dimensionless parameters, which is qualitatively similar to that of a single cylinder. Both upstream and downstream cylinders show smaller values of the individual and total drag coefficients than those for a single circular cylinder when these interact with each other.

Notations

$C_D = $ Total drag coefficient, dimensionless

$C_{D}^{\infty} = $ Normalized total drag coefficient using the corresponding Newtonian value, $[= C_D(\text{Non-Newtonian})/C_D(\text{Newtonian})]$, dimensionless

$C_{D}^{\infty} = $ Normalized total drag coefficient using the corresponding value for single cylinder value, dimensionless

$C_{DF} = $ Frictional component of the drag coefficient, dimensionless

$C_{DF}^{\infty} = $ Normalized friction drag coefficient using the corresponding Newtonian value, $[= C_{DF}(\text{Non-Newtonian})/C_{DF}(\text{Newtonian})]$, dimensionless

$C_{DF}^{SC} = $ Normalized friction drag coefficient using the corresponding value for single cylinder value, dimensionless

$C_{DF} = $ Pressure component of the drag coefficient, dimensionless

$C_{DF}^{\infty} = $ Normalized pressure drag coefficient using the corresponding Newtonian value, dimensionless

$C_{DF}^{SC} = $ Normalized pressure drag coefficient using the corresponding value for single cylinder value, dimensionless

$C_{DR} = $ Drag ratio, dimensionless

$C_{0} = $ Pressure coefficient, dimensionless

$C_{00} = $ Pressure coefficient at the front stagnation ($\theta = 0$) point, dimensionless

$C_{0}(\pi) = $ Pressure coefficient at the rear stagnation ($\theta = \pi$) point, dimensionless

$D = $ Diameter of the cylinders, m

$D_o = $ Outer boundary of the computational domain, m

$F_D = $ Drag force per unit length of the cylinder, N/m

$F_{DF} = $ Frictional component of the drag force per unit length of the cylinder, N/m

$F_{DF} = $ Pressure component of the drag force per unit length of the cylinder, N/m

$G = $ Gap ratio between the two cylinders, dimensionless

$I_2 = $ Second invariant of the rate of the strain tensor, s$^{-2}$

$L = $ Center-to-center distance between the cylinders, m

$m = $ Power-law consistency index, Pa.s$^n$

$n = $ Power-law flow behavior index, dimensionless

$\rho = $ Density of the fluid, kg/m$^3$

$\Psi = $ Stream function, dimensionless

$\tau = $ Shear stress, Pa

$x, y = $ x and $y$ components of the velocity, m/s

Greek Symbols

$\eta = $ Viscosity, Pa.s

$\theta = $ Angular displacement from the front stagnation ($\theta = 0$), degrees

$\rho = $ Density of the fluid, kg/m$^3$

$\Psi = $ Stream function, dimensionless

$\tau = $ Shear stress, Pa

$x, y = $ x and $y$ components of the velocity, m/s

Subscripts

$1 = $ Upstream cylinder

$2 = $ Downstream cylinder

$o = $ Free stream

Superscripts

$N = $ Newtonian value

$SC = $ Single circular cylinder

Literature Cited


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