FULLY DEVELOPED FLOW OF POWER-LAW FLUID THROUGH A CYLINDRICAL MICROFLUIDIC PIPE: PRESSURE DROP AND ELECTROVISCOUS EFFECTS

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ABSTRACT
Pressure drop and electroviscous effects in the axisymmetric, steady, fully developed, pressure-driven flow of incompressible power-law fluids through a cylindrical microchannel at low Reynolds number (Re = 0.01) have been investigated. The Poisson-Boltzmann equation (describing the electrical potential) and the momentum equations in conjunction with electrical force and power-law fluid rheology have been solved numerically using the finite difference method. The pipe wall is considered to have uniform surface charge density (S = 4) and the liquid is assumed to be a symmetric electrolyte solution. In particular, the influence of the dimensionless inverse Debye length (K = 2, 20) and power-law flow behaviour index (n = 0.2, 1, 1.8) on the EDL potential, ion concentrations and charge density profiles, induced electrical field strength, velocity and viscosity profiles and pressure drop have been studied. As expected, the local EDL potential, local charge density and electrical field strength increase with decreasing K and/or increasing S. The velocity profiles cross-over away from the charged pipe wall with increasing K and/or decreasing n. The maximum velocity at the center of the pipe increases with increasing n and/or increasing S and/or decreasing K. The shear-thinning fluid viscosity is strongly dependent on K and S, whereas the shear-thickening viscosity is very weakly dependent on K and S. For fixed K, as the fluid behaviour changes from Newtonian (n = 1) to shear-thinning (n < 1), the induced electrical field strength increases and maximum velocity reduces. On the other hand, the change in fluid behaviour from Newtonian (n = 1) to shear-thickening (n > 1) decreases the electrical field strength and increases the maximum velocity. The non-Newtonian effects on maximum velocity and pressure drop are stronger in shear-thinning fluids at small K and large S, the shear-thickening fluids show opposite influence. Electroviscous effects enhance with decreasing K and/or increasing S. The electroviscous effects show complex dependence on the non-Newtonian tendency of the fluids. The shear-thickening (n > 1) fluids and/or smaller K show stronger influence on the pressure drop and thus, enhance the electroviscous effects than that in shear-thinning (n < 1) fluids and/or large K where EDL is very thin.

NOMENCLATURE

- $D_i$: diffusivity of $i$th ionic species, m$^2$/s
- $E$: uniform induced electrical field strength, dimensionless
- $E_N$: normalized $E$, $E(N) = E(K, S, n)/E(K, S, n = 1)$
- $I_2$: second invariant of the rate of the strain tensor, s$^{-2}$
- $K$: inverse Debye length, dimensionless.
- $m$: power-law consistency index, Pa.s$n$.
- $n$: power-law flow behaviour index, dimensionless.
- $n_i$: ion concentration of $i$th ionic species, dimensionless
- $n^*$: charge density, $= (n_+ - n_-)$, dimensionless
- $\Delta P$: pressure drop, dimensionless
- $\Delta P_N$: normalized $\Delta P$ w.r.t. corresponding Newtonian value, $= \Delta P(K, S, n)/\Delta P(K, S, n = 1)$

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Over the past decades, enormous progress has been made in the development of technologies that pertain to miniaturization of conventional complex chemical analytical procedures into extremely small microchips considered as a subset of Micro-Electro-Mechanical Systems (MEMS), commonly referred to as Lab-On-Chips (LOC), Micro Total Analysis Systems (µTAS) or microfluidic chips. The key advantages of microfluidic devices include the significantly reduced quantity of sample consumption and increased rate of heat/mass transfer and chemical reactions. The rapid growth of novel and complex microfluidics devices has mainly been motivated by the fast development in MEMS-related fluidic devices ranging from pH and temperature sensors to fluid actuators, such as micronozzels, pumps, mixers, and valves [1], biotechnology areas such as the analysis of DNA and proteins, and biodefence [2], as well as LOC systems for drug delivery, chemical analysis, and biomedical diagnosis [3]. A blood analyzer MEMS requires taking a sample by means of generating blood microflow through a microneedle of the microfluidic system.

The above-mentioned applications require the ability to transport, manipulate and process fluids (in general aqueous based solutions) through micro-channels. Microfluidic flows can be driven by pressure or electrical potential difference. For the optimal design of the new microfluidic systems, it is important to understand the microchannel flow characteristics, such as pressure distribution, heat transfer, and transport behaviour of the flow. However, the relative importance of the forces that can influence fluid flow is different at the length scales of these devices (typical characteristic length ~ 10 to 200 µm). For example, as the size of the device is reduced, the importance of surface-based phenomena, such as surface tension and electrokinetic effects, increases [1, 4, 5].

Most solid surfaces bear electrostatic charges. When an ionic liquid is brought into contact with a charged surface, electrokinetic phenomena develop as the surface charges attract counter-ions in the liquid. The rearrangement and balancing of the charges gives rise to a region called the electrical double layer (EDL) [6, 7]. The EDL is the region of variation in electric potential away from the surface. The presence of an EDL in the pressure-driven flow of ionic liquids results in an extra hydrodynamic resistance (electroviscous effect). This occurs because ions are initially transported by the pressure driven flow. These ions accumulate at the walls of the channel, which induces an electrical field along the length of the channel (streaming current) that resists fluid flow [5–8].

Many substances of multi-phase nature and/or of high molecular weight encountered in industrial practice (pulp and paper suspensions, food, polymer melts, solutions and in biological process engineering applications, etc.) display shear-dependent viscosity behaviour [10]. In such fluids, the viscosity either increases (shear thickening) or decreases (shear thinning) with increasing shear rate. Owing to their high viscosity levels, these materials are generally processed in laminar flow conditions. In microfluidic devices, the small dimensions suppress the development of turbulence and fluids flow in laminar conditions. Additionally, due to the microscopic size of the channel, microfluidic flow shows an another important feature of modifying the rheology of simple Newtonian fluid into the non-Newtonian fluid. These non-Newtonian effects are expected to be important for polymeric liquids and particle suspension flow [11]. Microfluidics is also a promising approach for the investigation of the rheology of non-Newtonian fluids within confined microenvironments [12].

Adequate information associated with the Newtonian flow through microfluidic geometries is available in the literature, e.g., see [13, 14] for the short review on electrokinetic flow through rectangular and cylindrical microchannels. Most of the available studies on cylindrical microchannels either assume that the double layer is very thin or related to electro-osmosis effects [7]. Much less is known about the electrokinetic effects in non-Newtonian flow through cylindrical microchannels. Among others, Girardo et al. [12] investigated the rheological properties of a non-Newtonian liquid within microchannel in the temperature range above the vitrification region. Subsequently, Zimmerman et al. [15] carried out 2-D finite element simulations of a electrokinetic flow of Carreau fluids in a T-junction microchannel to demonstrate a potential design of electrokinetic flow rheometer in a microchannel. Srivastava and Burns [16] used a microfluidic capillary viscometer to analyze non-Newtonian power-law fluids. More recently, Barkhordari and Etemad [17] performed numerical simulations to obtain the steady flow and thermal fields...
of non-Newtonian power-law fluids in circular microchannels. However, there has been no prior study dealing with electroviscous effects in the steady flow of power-law fluids through electrically charged uniform cylindrical microchannel. Thus, the present study aims to investigate the electroviscous effects in conjunction with power-law rheology in steady fully developed flow through a cylindrical microchannel.

MATHEMATICAL FORMULATION

Consider the axisymmetric, steady, laminar, fully developed, pressure-driven flow of an incompressible ionic liquids through a cylindrical microchannel (of radius, $R$) (Fig. 1). The pipe wall is considered to have uniform surface charge density $\sigma$. The dielectric constant ($\varepsilon$) and density ($\rho$) are assumed to be spatially uniform. Furthermore, it is assumed that the liquid contains symmetric anions and cations (specified by + and -, respectively) with equal valencies ($z_+ = z_- = z = 1$) and equal diffusivities ($D_+ = D_- = D$). The bulk ion concentration of each ion species is $n_o$. The rheology of the fluid is governed by the non-Newtonian power-law fluid model. The rheological equation of state for incompressible fluids is given by

$$\dot{\gamma} = 2\eta'\dot{\gamma}' \quad \text{where} \quad \dot{\gamma}' = \frac{\nabla\dot{\gamma} + (\nabla\dot{\gamma})^T}{2}$$

(1)

where $\dot{\gamma}'$ is the rate of strain tensor. For power-law fluids, the viscosity ($\eta'$) is given by

$$\eta' = m(2I_2)^{(n-1)/2} \quad \text{and} \quad I_2 = (\nabla : \dot{\gamma}')$$

(2)

where power-law constants $m$ and $n$ are the consistency index and the flow behaviour index of the fluid ($n < 1$: shear-thinning; $n = 1$: Newtonian; and $n > 1$: shear-thickening) and $I_2$ is the second invariant of the rate of the strain tensor ($\dot{\gamma}'$).

The electroviscous flow of ionic liquids is described by the equation of continuity and the Navier-Stokes equations with an electrical body force term. These flow field equations are coupled with the Poisson-Boltzmann equation relating the electrical potential to the charge distribution. The field equations have been rendered dimensionless using $R$, $\nabla$, $R/\nabla$, $p\nabla^2$, $m(\nabla/R)^n$, $m(\nabla/R)^{n-1}$, $n_o$ and $k_b T/ze$ as scaling variables for lengths, velocities, time, pressure, stress components, viscosity, ion concentrations and electrical potential, respectively. The non-dimensionalization using these scaling parameters results in the following dimensionless groups:

$$Re = \frac{\rho V^2 R^m}{m}, \quad Sc = \frac{m \rho D (\nabla R)^{n-1}}{\varepsilon_0 \sigma R}, \quad S = \frac{ze^2 R}{\varepsilon_0 k_b T}$$

$$B = \frac{\rho k^2 T^2 e^2}{2 (ze^m)^2} \left(\frac{\nabla}{R}\right)^{2(1-n)}, \quad K = \sqrt{\frac{2ze^2 n_0 m^2 R^2}{\varepsilon_0 k_b T}}$$

Here, $Re$ and $Sc$ are the Reynolds and Schmidt numbers, respectively. $S$ and $K$ are the dimensionless surface charge density and the dimensionless inverse Debye length (i.e., proportional to the ratio of the pipe radius to the EDL thickness), which depends on the bulk ion concentration ($n_o$), and $B$ is a dimensionless parameter which depends on the liquid properties at a specified temperature. Here, $e$, $k_b$, $T$, $\varepsilon$ and $\varepsilon_0$ are the elementary charge, the Boltzmann constant, temperature, dielectric constant of the solution and the permittivity of the free space, respectively. Under the above-noted simplified assumptions, the dimensionless governing equations for the electroviscous flow through cylindrical microchannel are given as follow:

Electrical Double Layer (EDL) Potential Field

According to the theory of electrostatics, for a symmetric $(z : z)$ electrolyte solution, the relationship between the total electrical potential ($U$) and the net charge density ($\rho_e$) at any point in the liquid is described by the Poisson equation, as follow:

$$\nabla^2 U = K^2 (n_+ - n_-)/2 \quad \text{where} \quad U = \psi(r) - Ex$$

(3)

where, $E(= -dU/dx)$ is the uniform induced electric field strength along the uniform pipe, and $\psi$ (the EDL potential) is related to the ion concentration in a symmetric electrolyte solution by the equilibrium Boltzmann distribution as follow:

$$n_+ = e^{-\psi} \quad \text{and} \quad n_- = e^{\psi}$$

(4)
where, \( n_+ \) and \( n_- \) are the numbers (per unit volume) of anions and cations, respectively. For the fully developed electric field, the EDL potential \( \psi \) satisfies the Poisson-Boltzmann equation:

\[
\frac{1}{r} \frac{d}{dr} \left( r \frac{d\psi(r)}{dr} \right) = K^2 \sinh \psi(r) \quad (5)
\]

**Boundary Conditions:** The EDL potential field is subjected to the following boundary conditions: uniformly charged microchannel wall and symmetric at the centerline.

\[
\left. \frac{d\psi}{dr} \right|_{r=0} = 0 \quad \text{and} \quad \left. \frac{d\psi}{dr} \right|_{r=1} = S \quad (6)
\]

**Flow Field Coupled with Electrokinetic Interaction**

For axisymmetric, steady, fully developed flow: \( V_s = V_s(r), V_r = 0 \) and both the unsteady term and inertia term \((V_rV_r)\) vanish in the momentum equation. Also, the axial pressure gradient \((dP/dr)\) is constant and uniform. Thus, the equation of motion for an incompressible fluid can be written as

\[
0 = -\frac{dP}{dx} + \frac{1}{Re} \left[ \frac{1}{r} \frac{d}{dr} \left( r \tau_{rr} \right) \right] + F_s \quad (7)
\]

In absence of the gravitational field, the body force per unit volume \((F_s, \text{ dimensionless})\) caused by the induced electric field \((E)\) in the EDL region is given by \( F_s = B(K/Re)^2(n_+ - n_-)E \).

Now, let \( \alpha = Re(dP/dx), V_s^* = V_s/\alpha \) and \( E^* = E/\alpha \) and Eq. \((7)\) can be written as

\[
0 = -1 + \left[ \frac{1}{r} \frac{d}{dr} \left( r \eta \frac{dV_s^*}{dr} \right) \right] - \frac{2BE^*K^2}{Re} \sinh \psi(r) \quad (8)
\]

where \( \eta = (2I_2)^{(n-1)/2} \) is the dimensionless viscosity of power-law fluids.

**Boundary Conditions:** This electroviscous flow is subjected to the following conditions: No-slip microchannel wall and symmetric at the center of channel.

\[
\left. \frac{dV_s}{dr} \right|_{r=0} = 0 \quad \text{and} \quad \left. V_s \right|_{r=1} = 0 \quad (9)
\]

For the fixed volumetric flow rate \((Q)\) and the dimensionless average velocity equals unity \((\bar{V} = 1)\), we have

\[
2 \int_0^1 rV_s dr = 1 \quad \text{and hence} \quad \alpha = \left( 2 \int_0^1 rV_s^* dr \right)^{-1} \quad (10)
\]

During steady-state pressure driven electroviscous flow, the net axial current \((I_{\text{net}} = I_s + I_d + I_c)\) is assumed to be zero. Hence,

\[
2\pi \int_0^1 \left[ \frac{1}{Pe} \frac{1}{I_d} \frac{dI_+}{dx} - \frac{1}{Pe} \frac{1}{I_s} \frac{dI_-}{dx} \right] \frac{1}{I_c} \frac{dP}{dx} \quad (11)
\]

where \( Pe = (ReSc) \) is independent of the power-law constants. Here, \( I_s, I_d, I_c \) are the streaming, diffusion and conduction currents, respectively. Under steady fully developed flow condition, the diffusion current \( I_d = 0 \) (eq. 10). After substituting \((dU/dx) = -E\) and \( n_\pm = e^{\mp\psi} \), the electric field strength \((E)\) is given by the following expression:

\[
E = Pe \left( \frac{\int_0^1 rV_s \sinh \psi dr}{\int_0^1 r \cosh \psi dr} \right) \quad \text{or} \quad E^* = Pe \left( \frac{\int_0^1 rV_s^* \sinh \psi dr}{\int_0^1 r \cosh \psi dr} \right) \quad (12)
\]

**NUMERICAL SOLUTION METHODOLOGY**

Owing to the axial symmetry of the flow, the solution is obtained only in the one symmetric half \((r \geq 0)\) of the microchannel. The field equations and boundary conditions are discretized using a finite difference method (FDM). The numerical solution is obtained in two steps. As the EDL potential field is independent of the flow field, a iterative solution of the EDL potential field equations (eqs. 4-6) is used to yield the EDL potential, \( \psi(r) \), and ion concentration \((n_+, n_-)\) fields. Then, a iterative solution of the flow field equations (eqs. 2, 8-12) yields the velocity \((V_s(r))\) field, pressure drop \((dP)\) and the electrical field strength \((E)\). A grid independence study has been carried out with three different grids (grid spacing, \( \Delta r \)) for \( K = 2, 20; S = 4; n = 0.2, 1, 1.8 \). The results do not change (maximum deviation being less than 0.5%) in moving from \( \Delta r = 0.005 \) to \( \Delta r = 0.002 \). However, the grid \( \Delta r = 0.002 \) is used to obtain the new results.

**RESULTS AND DISCUSSIONS**

Numerical computations have been carried out for a symmetric electrolyte solution \((B = 2.34 \times 10^{-4} \text{ and } Sc = 10^2, \text{ based on the properties of water at } T = 298K) \) at low Reynolds number \((Re = 0.01)\) flow. In particular, the influence of the dimensionless inverse Debye length \((K = 2, 20)\) and power-law index \((n = 0.2, 1 \text{ and } 1.8)\) for surface charge density \( S = 4 \) on the velocity, ion concentrations and electrical potential fields, pressure drop and uniform induced electrical field strength have been studied. The results have also been obtained for non-electrokinetic flow \((S = 0 \text{ or } K = \infty)\). Only positive values of \( S \) are considered since the

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results for negative $S$ can be obtained by setting new values of $U = -U$ and $n_+ = n_-$. However, prior to presenting new results, the validation of the present solution procedure is presented here to ascertain the accuracy of the new results.

Validation - Non-Electroviscous Pipe Flow of Power Law Fluids: The accuracy of the present numerical method is confirmed by comparing the maximum velocity for non-electroviscous flow ($S = 0$) of power-law fluids in a cylindrical pipe against the analytical results, $V_{max} = (3n + 1)/(n + 1)$. The results are summarized in Table 1.

<table>
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<tr>
<th>power-law index ($n$)</th>
<th>Numerical $V_{max}$</th>
<th>Analytical $V_{max}$</th>
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<td>1.3333</td>
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<td>2.0000</td>
</tr>
<tr>
<td>1.8</td>
<td>2.2864</td>
<td>2.2857</td>
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</table>

EDL Potential Distribution: Figure 2 shows the variation of EDL potential ($\psi$) along the radius ($r$) of the pipe for $K = 2, 20$ and $S = 4$. Due to the positively charged pipe wall, as expected, the potential is highest near the wall and decreases to the lowest value at the center of the pipe. The EDL potential becomes greater as $K$ decreases since the EDLs then occupy a greater fraction of the pipe radius exposing more counterions to the flow.

Increasing $K$ reduces the EDL thickness and decreases the potential in the inner region of pipe. The zero potential (except that close to wall) in the pipe for $K = 20$ shows that the EDL effects in this case are confined to the thin region near the wall and the flow will behave like the non-electroviscous ($S = 0$ or $K = \infty$) case.

Charge Distribution: The difference between the counter-ion and co-ion number concentration is referred to as the excess ionic number concentration ($n^* = n_+ - n_-$). For symmetric ($z:z$) electrolyte, the dimensionless net excess charge is proportional to $n^*$. Figure 3 shows the dimensionless charge ($n^*$) distribution along the radius ($r$) of pipe. Generally, the net charge magnitude increases as $K$ decreases (EDL thickness increases). Due to the positive surface charge ($S$), the charge density ($n^*$) is higher in the region close to the wall. Both decreasing $K$ (thickening of EDL) and an increasing positive surface charge density ($S$) enhances the negative ion concentration.

Velocity Profiles: Figures 4 and 5 show the effect of $K$, $S$ and $n$ on the fully developed velocity ($V = V_0(r)$) through a positively charged pipe. The velocity profiles in electroviscous flows are qualitatively similar to that in non-electroviscous flow, i.e., for fixed $n$, $V$ is decreases from its maximum value at the centerline ($r = 0$) toward the wall ($r = 1$) through a positively charged pipe. The influence of power-law index ($n$) on the velocity profiles is qualitatively similar for both electroviscous ($S = 4$) and non-electroviscous ($S = 0$) flow. At the smaller value of $K$ in electroviscous flows (where there is a very thick EDL), the local velocity of shear-thinning and Newtonian fluids ($n \leq 1$) is greater in the middle region ($r < 0.7$) of the pipe and smaller close to the pipe wall than that in non-electroviscous flow. The deviation between the
Corresponding Newtonian values, i.e., shear-thickening \( n > 1 \), ever greater for shear-thinning \( n < 1 \) (and \( n > 1 \)) to Newtonian \( (n = 1) \) and then to shear-thickening \( (n > 1) \). The electroviscous effects are, however, greater for \( n < 1 \) than that for \( n \geq 1 \).

In order to elucidate the influence of power-law rheology, flow variables have been normalized with respect to the corresponding Newtonian values, i.e., \( \phi_N = \phi(n)/\phi(n = 1) \), where \( \phi = V, V_{\text{max}}, E, \Delta P \), etc., under otherwise identical conditions. Figures 6 and 7 show the dependence of \( V_N \) on \( K, S \) and \( n \). The qualitative variation of \( V_N \) is similar in both electroviscous and non-electroviscous flow, except at \( K = 2 \) near the wall in the shear-thinning fluid \( (n = 0.2) \). The normalized maximum velocity \( (V_{\text{max},N}) \) is larger (and smaller ) for \( n < 1 \) (and \( n > 1 \)) in electroviscous flow than that in non-electroviscous flow. Both figures show stronger influence of power-law rheology (especially shear-thinning fluids) with decreasing \( K \). These influence are due to the complex dependence of the power-law fluid viscosity (see Fig. 8) in electroviscous flow.

**Viscosity Distribution:** Figure 8 shows the effect of \( K \) and \( S \) on the dimensionless viscosity (\( \eta \)) of power-law fluids under fully
developed flow conditions. The shear-thickening \((n > 1)\) fluid viscosity is nearly independent on the electroviscous flow parameters \((K\) and \(S)\). The viscosity of the shear-thinning \((n < 1)\) fluids almost monotonically decreases from its maximum value at the center of pipe toward the pipe wall. In non-electroviscous flow, viscosity reduces until the pipe wall. For \(K = 2\), a slight increase in viscosity is seen near the wall. The shear-thinning viscosity is smaller for electroviscous flows than for non-electroviscous flow, except near the wall where there is a cross-over in the viscosity for electroviscous and non-electroviscous flows. It is due to the thickening of EDL, which occupies the greatest cross-section of the pipe at the smallest value of \(K\). This exposes more counterions to the flow which results in the stronger potential gradient (see Fig.2) and electrical forces are stronger even far away from the wall. In addition, for shear-thinning \((n < 1)\) fluids, the viscosity \((\eta)\) becomes very large as the shear-rate decreases and hence it tends to infinity when the shear rate is almost zero. Therefore, viscous effects dominate even far away from the wall. Conversely, for shear-thickening \((n > 1)\) fluids, viscous effects are very weak.

**Electrical Field Strength:** Figures 9 and 10 show the effect of \(K\), \(n\) and \(S\) on the induced electric field strength \((E)\). For a fixed \(n\), the induced electric field strength \((E)\) increases with increasing \(S\) and/or decreasing \(K\). At higher \(K(= 20)\) (very thin EDL), \(E \approx 0\) and thus the influence of electric field on the flow behaviour will be very small. In this case, the flow will behave as the non-electroviscous \((E = 0 \text { at } S = 0 \text { or } K = \infty)\) flow. The electric field strength \((E)\) weakens with increasing \(n\). Figure 10 shows that the effect of power-law rheology is stronger at higher \(K\) in shear-thinning \((n < 1)\) fluids and smaller \(K\) in shear-thickening \((n > 1)\) fluids.

**Pressure Drop:** The influence of \(K\), \(S\) and \(n\) on the local flow characteristics should also be reflected on the pressure drop \((\Delta P)\). Figures 11 and 12 show the effect of \(K\), \(n\) and \(S\) on \(\Delta P\) in electroviscous flow of power-law fluids through a pipe. For a fixed value of \(n\), \(\Delta P\) increases with increasing \(S\) and/or decreasing \(K\). It decreases as the fluid behaviour changes from Newtonian \((n = 1)\) to shear-thinning \((n < 1)\) fluids, however, the opposite is seen in shear-thickening \((n > 1)\) fluids. As a consequence of stronger viscous effects in shear-thinning \((n < 1)\) fluids, the pressure drop increases with a increasing shear-thinning behaviour. The opposite is true in shear-thickening fluids increase the pressure drop. Figure 12 shows that the electroviscous effects are stronger in...
shear-thickening fluids at higher $K$, however, opposite is seen in shear-thinning fluids.

**Electroviscous Effects:** In pressure-driven electrokinetic flow, the induced (streaming) axial potential gradient produces an additional hydrodynamic resistance to the flow (i.e., electrical force in Eq. 7). For a fixed volumetric flow rate ($Q$), this additional resistance manifests itself as a pressure drop ($\Delta P$) along the pipe is higher than the pressure drop in absence of the electroviscous effects ($\Delta P_0$). Thus, the electroviscous effect is often quantified in terms of an apparent (or effective) viscosity. The apparent viscosity ($\mu_{eff}$) is the viscosity of the fluid (in absence of electroviscous forcing) required to achieve the pressure drop ($\Delta P$) observed in presence of the electroviscous forcing. For low $Re$ steady flow, the non-linear convection term in the momentum equation is small, and electroviscous effect can be represented in terms of the electroviscous correction factor ($Y$) related to the apparent viscosity and pressure drop are related via:

$$ Y = \frac{\mu_{eff}}{\mu} = \frac{\Delta P}{\Delta P_0} $$

where, $\mu$ is the physical viscosity of the liquid and $\Delta P_0$ is the pressure drop in non-electroviscous ($S=0$) flow. The electroviscous correction factor ($Y$) is the quantitative measure of electroviscous effects. Figure 13 shows the variation of electroviscous correction ($Y$) to the pressure drop in power-law fluids. Similarly, Figure 14 shows the electroviscous effect on the normalized maximum velocity ($V_{max,S} = V_{max}/V_{max,0}$).
here $V_{\text{max,0}}$ is the maximum velocity in non-electroviscous flow) through a pipe. As expected, the normalized values exceeds 1, over the range of conditions. The $Y = 1$ represents the liquid flow with no-electrokinetic effects. For Newtonian flow, these trends are qualitatively similar to that reported in the literature for uniform microchannels [14]. At large $K$, both normalized maximum velocity ($V_{\text{max},S} \approx 1$) and electroviscous correction factor ($Y \approx 1$) are seen to be almost independent of the power-law index ($n$). It represents that the electroviscous effects are negligible at $K = 20$. The electroviscous effects enhance with decreasing $K$ and/or increasing $S$ in both shear-thinning and shear-thickening fluids. However, the electroviscous effects are stronger (Figs. 14 and 13) in shear-thinning ($n < 1$) fluids than that in Newtonian and shear-thickening ($n > 1$) fluids.

In summary, the fully developed flow characteristics of power-law fluids in a electrically charged microfluidic cylindrical pipe are influenced in an intricate manner by the value of dimensionless inverse Debye length ($K$), surface charge density ($S$) and the power-law index ($n$). The electroviscous effects are more prominent at small $K$, small $n$ and large $S$. This interplay is further accentuated by the fact that the viscous term in the momentum equation is highly non-linear function of power-law index for power-law fluids. At low Reynolds number, the viscous forces approximately scales as $V^n$ and, thus, keeping everything else fixed, the viscous effects are likely to grow with the increasing $n > 1$. These non-linear interaction in conjunction with electrical effects exert a strong influence on the pressure drop and fully developed velocity profile.

CONCLUSIONS

Electroviscous effects in axisymmetric, steady, fully developed pressure-driven flow of ionic (symmetric 1:1 electrolyte) liquid in a cylindrical microfluidic pipe at low Reynolds number ($Re = 0.01$) are investigated numerically by solving the Poisson-Boltzmann the Navier-Stokes equations in conjunction with electric field and power-law fluid rheology for the flow field using a finite difference method. Numerical results for the EDL potential, ion concentrations, charge distribution, induced electrical field, fully developed velocity and pressure drop are presented and discussed over the following ranges of conditions: dimensionless inverse Debye length $K = 2, 20$ and power-law index, $n = 0.2, 1, 1.8$ at Schmidt number, $Sc = 1000$ and liquid specified parameter, $B = 2.34 \times 10^{-4}$. The pipe walls are considered to have a uniform surface charge density ($S = 4$).

Over the range of parameters, the electroviscous effects are negligible at $K = 20$. As expected, the local EDL potential, local charge density and electrical field strength increases with decreasing $K$ and/or increasing $S$. The velocity profiles cross-over away from the charged pipe wall with increasing $K$ and/or decreasing $n$, i.e., lower velocity at small $K$ and large $n$ close to the wall and opposite away from the wall. The maximum velocity at the center of the pipe increases with increasing $n$, with decreasing $K$, and with increasing $S$. The shear-thinning fluid viscosity show a strong dependence on $K$ and $S$, whereas the shear-thickening viscosity is very weakly dependent on $K$ and $S$. For fixed $K$, as the fluid behaviour changes from shear-thinning ($n < 1$) to Newtonian ($n = 1$), and then to shear-thickening ($n > 1$), the induced electrical field strength weakens the electrical field strength and the maximum velocity increases. The non-Newtonian effects on maximum velocity and pressure drop are stronger in shear-thinning fluids at small $K$ and large $S$ while the shear-thickening fluids show opposite dependence. Electroviscous effects increase with decreasing $K$ and/or increasing $S$. The electroviscous effects show complex dependence on the non-Newtonian tendency of the fluids. The shear-thickening ($n > 1$) fluids and/or smaller $K$ show stronger influence on the pressure drop and thus, enhance the electroviscous effects than that in shear-thinning ($n < 1$) fluids and/or large $K$ where EDL is very thin.

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REFERENCES


