

# **An Interview with Alain Connes**

## **G. B. Khosrovshahi (GBK)**

Our first question is what do you think will be the main trends of mathematics in the 21th century.

## **Alain Connes (C)**

Well, thankfully the development of mathematics is not something one can predict, and it would be foolish to try. One reason we love doing mathematics is that we don't know what lies ahead that future research will uncover. It is however possible to explain specific examples of mysterious structures that we need to understand better. I was recently asked to give a talk on the "challenges of the twenty first century" in mathematics, and rather than trying to give a long list I focussed on just 2 examples of spaces which are easy to introduce but whose geometry is still quite mysterious. The first is space-time, the second is the space of prime numbers. I explained in my four talks here very small fragments of their geometry but obviously we'd like to know a lot more!

## **M. Khalkhli (MK)**

Some people say that the 21 century will be the century of informatics and biomathematics and these two activities will dominate the 21 century. Don't you agree with this?

## **C**

It is clear that computers are of great help to mathematicians and that at this moment biomathematics attracts a lot of money because of its potential

applications. There is interesting work done in biomathematics and mathematics is really needed as a tool to cope with this enormous amount of information given for instance by the genome. Some of the potential applications are really nice but this does not suffice to view it as a dominating activity of mathematics, except at the level of the amounts of money involved which means nothing. As far as informatics goes it is clear that computers are great tools and that they change our perception of the ranking of various domains of mathematics. For instance it used to be a precious skill to have complete control of differential equations, special functions etc... but nowadays computers can do these things much better. You just enter the equation and write `Dsolve`, that's it, you get the answer... This in retrospect gives a lot more value to domains of mathematics that could before be considered as too abstract.

**GBK**

Do you think you will use computers in your mathematical work?

**C**

Well, you know I have used computers very heavily in recent years.

**GBK**

really?

**C**

Yes! I got over the years two different perceptions of computers. What happened first, several years ago, was when we were computing the local index formula for foliations with Henri Moscovici. At that point we had a

negative perception of computers. We were thinking it was much better to do computations by hand than to do them using a computer. So we spent separately 3 weeks 8 hours a day to compute some formula which we wanted to know. After that time of course we compared our results and we found there were minor mistakes. After these minor corrections our results agreed but we were supposed to get a cocycle and the formula didn't give us a cocycle.

Then we got really worried that perhaps there was a mistake in the theory but we found by staring at the result that if we change the sign of 8 of the 36 terms appearing in the final formula then it became a cocycle. We first went back to check again these terms and realized after a while that the reason why their signs were wrong was quite subtle : we had made a conceptual mistake in doing the computation and had forgotten some crucial terms in the differential operators below the sub-principal symbol! At that point I convinced myself that we could never find this subtle correction without the intimate knowledge of all the terms of the formula which only the slow computation "by hand" could give us...

Several years later I changed my mind. Namely I met in my work with Michel Dubois-Violette a result that was expressed as the sum of 1440 integrals, each the integral over a period of an elliptic curve (with modulus  $q$ ) of a rational function of high degree in theta functions and their derivatives. Even in the trigonometric limit it was difficult to compute this sum. So we used the computer in that limit case first and it produced a beautiful formula in terms of the parameters of the theory. Then it took us about 6 months to guess how to extend this formula in the elliptic case using Jacobi's elliptic functions  $\text{sn}$   $\text{cn}$  etc...but there was still an unknown function of the modulus  $q$  in front which we could not guess. We then found some simplifications in the theory which divided the computing time by a factor of fifty and then we got the first terms in the expansion in powers of  $q$  and recognized the first

terms in the expansion of the ninth power of the Dedekind eta function. We could predict the next term but it was still beyond the computing power of a private computer and we had to use the computer system in école polytechnique. It produced exactly the predicted term. We were then sure that we had the correct formula. It then took a lot of hard work to understand its meaning but we eventually did it.

So then I cannot deny that computers can be incredibly useful because it was simply not possible to do things by hand in that case: each of the 1440 integrals, when expanded in powers of  $q$  up to the relevant power, was taking up to 200 pages of trigonometric formulas. You can't do this by hand; it's just impossible. Clearly computers make it possible to see much further in some circumstances! I view them as a great help, a bit like a slave doing without mistakes and complaints the most tedious tasks ever!

## **GBK**

Computers may not like this label "slave".

## **C**

In that case computers were very useful and I kept using them later on to check for instance some involved algebra computations involving Hopf algebras etc... It will never replace human thinking at the level of conceptual problems, analogies, and things like that, but as a tool it is fantastic. So it is like an extremely good pair of glasses, I like it and have nothing against it. I have heard some people say that computers can only check finitely many cases but this is totally wrong because the computer does formal computations like simplifying trigonometric expressions, etc. They can even do logical deductions, you give them a set of axioms and logical rules and they can prove simple results.

**GBK**

You know there are algorithms to check the proofs.

**C**

But at the moment they are very clumsy and they do the checking in a very mechanistic formal manner which is not what we'd like to have. It will come but it will take time to make them efficient and user friendly in that respect. But to check computations they are already unbeatable!

**GBK**

Do you think quantum computers will be realized one day?

**C**

The main problem with these quantum computers is that they have to be implemented "physically". It is a very hard practical problem of experimental physics. It's not a theoretical problem and I think what is quite hard indeed is to prepare the system and read the result, thus passing from quantum to classical. Of course the applicability of the whole theory depends on that practical implementation. Experimenters tend to do miracles so we just need to wait and see.

**GBK**

I want to ask you a question. What do you think about string theory?

## C

Well string theory has uncovered beautiful relations between physics and different parts of mathematics, mostly differential geometry, enumerative algebraic geometry and complex analysis. In fact string theory started like this. At the very beginning when Veneziano and others were starting string theory motivated by the dual resonance model for strong interactions they found solutions to the duality equations for the scattering amplitudes in terms of Mandelstam parameters and these were nice mathematical functions like generalized beta functions which are very natural in terms of complex analysis. It was then realized, by Suskind and others, that these models could be understood geometrically from the propagation of strings. It is a very powerful idea to “test” a given complex space using the space of complex curves inside or of maps from Riemann surfaces to that target space. And physicists could use all the arsenal of conformal field theory which is quite powerful. This generated a very interesting group of people that do a kind of “physics motivated” mathematics which rejuvenated some parts of complex geometry. They adopt a rather free attitude towards mathematics, which is original and productive and had a very positive influence. It started as mathematics and had a very positive impact on mathematics up to now.

There is a well known anecdote of Pauli attending a lecture, von-Neumann was at the blackboard proving a theorem, and Pauli interrupted him saying “If doing physics were proving theorems you would be a great physicist”. Fine but it is not enough to be careless about proofs to really qualify as a physicist. The true question is whether or not string theory has anything to do with reality and we all know that this is a key question. That key question begins by supersymmetry; whether or not nature is supersymmetric, whether or not for example there is a Photino which is a fermion and is the super partner of the photon, and whether or not for each quark there is a squark which is a force which is the super partner of that quark, a force that so far has not been

felt. If one would already have found 3 or 4 super partners by now, then I would believe they would find the others but in reality they haven't found any and because of that I'm very skeptical. In ancient times there was a theory saying that in the solar system there is a planet which is the same as the earth but which we cannot see because it is exactly symmetrical with respect to the sun. I don't feel that supersymmetry is far more convincing than that theory. The supersymmetric standard model is an horrible thing...with more than a hundred free parameters and an ugly mechanism to break that "beautiful" unseen supersymmetry! Until I am shown more evidence I have the attitude of a skeptic. We'll see in 2007, might be some convincing experimental result will be found, but till then I remain unconvinced, even by the very starting point of string theory.

## **GBK**

But you admire their mathematics

## **C**

I do for sure. If I did not I would be cutting the branch on which I am sitting. For example they use noncommutative geometry, they do beautiful things and their expertise in physics is great but still I remain unconvinced at the moment that nature has chosen this path. I think it's very important to construct other competing models which are not necessarily based on supersymmetry. I think it is crucial for the development of physics that there are people who are courageous enough not to follow the main dogma, heretics that develop a different model.

## GBK

Do you think there is a perfect model for physicists in horizon? They are all thinking about string theory.

## C

Particle physics contains treasures which have been tested with great precision. This includes both the renormalization technique and the standard model. I respect these great discoveries and spent a great deal of time to understand both at the conceptual level. For renormalization the conclusion of my work with Dirk Kreimer and then with Matilde Marcolli is that the divergences are in fact a blessing since they are the generators of the action of this marvelous cosmic Galois group that Cartier had guessed. The lesson was very hard to learn but what appears is that our naive idea of a four dimensional continuum needs to be approached from nearby complex dimensions using a specific universal method that in essence renormalizes the geometry. What is quite intriguing is that this method matches perfectly with the noncommutative correction of space-time which yield a description of the standard model simply as the inner part of gravity. With these lessons well understood the next step is to combine the spectral framework of geometry provided by NCG with the Feynman recipe of summing over all geometries. The starting point is to extend the classical diffeomorphism invariance to the point that the only observables are “spectral” i.e. measure spectral invariants of a geometry. One can then write a functional integral over all geometries, including noncommutative ones, and it gives a certain matrix model. The remaining problem of fixing the constraints that single out “Dirac operators” among Hermitian matrices was the main motivation behind the work with Dubois-Violette which I alluded to above.

There is also loop gravity for quantum gravity but I’m not an expert.



I don't see how they can get around renormalization in what they do but for sure there is something nice to say for spin networks. It is important that different approaches be developed and that one doesn't try to merge them too fast. For instance in noncommutative geometry my approach is not the only one, there are other approaches and it's quite important that for these approaches there is no social pressure to be the same so that they can develop independently. It's too early to judge the situation for instance in quantum gravity. The only thing I resent in string theory is that they put in the mind of people that it is the only theory that can give the answer or they are very close to the answer. That I resent. For people who have enough background it is fine since they know all the problems that block the road like the cosmological constant, the supersymmetry breaking, etc etc...but if you take people who are beginners in physics programs and brainwash them from the very start it is really not fair. Young physicists should be completely free, but it is very hard with the actual system.

## **GBK**

And how far they can go in their mathematics? You said that string theorists are much more mathematical than physicists. How far they can go in their mathematics?

## **C**

In mathematics as I said above they had a great impact in complex geometry. Superstring theory gives you a ten dimensional space which has six dimension more than our standard observed space-time. The simple idea close to Kaluza-Klein is that you have a fibration with very tiny Planck size six dimensional spaces which one looks for among Calabi-Yau manifolds to comply with the basic constraints of the vacuum states of the theory. So in

short they do three dimensional complex geometry by looking at how complex curves propagate in such a space. They are mathematicians but they are mathematicians with a different type of attitude. They are very free and work in extremely smart packs, they are not hunting for the same things, and have tools that complex geometers did not have so they were able to produce great stuff.

As far as physics goes the situation is totally different since the dream was that there would be an essentially unique vacuum i.e. a Calabi-Yau manifold singled out by string theory constraints and that it would just reproduce the phenomenology of the Standard model of particle physics. After twenty years that goal looks much further remote and appears like a mirage. I think the real thing which bothers me is that what now is propagated in the public is the idea that quantum gravity exists, it is string theory and the universe is made of strings and you read this in popular books which give this idea and people take it without any reservation. Many people do believe that the universe is made of strings. What evidence do we have for that? Nothing!

## **GBK**

In a place like here (IPM) which we have started everything from scratch, our physics department is totally dominated by string theory and it's not good. it's not healthy for physics.

## **C**

The string theory group here is just marvelous with Ardanan as a great guide! And they did catch up very early with NCG and did beautiful contributions. The problem with physics in the long run is that the actual trend is to put all your eggs in one basket which is probably not very safe. Indeed most theoretical physicists do string theory and I don't think this is completely

healthy for physics in the sense that you know.

**GBK**

Do you think there are many people doing string theory? I've heard that there are only 700 active people in string theory in the world.

**C**

There was a string theory conference in Paris in 2004 which attracted between 600 and 700 people.

**GBK**

So the number is not that low.

**C**

I would say there are 2000 people working in string theory while in NCG I would say there are about 150 to 200 people.

**GBK**

An string theory group does not cost much, it's like mathematics.

**C**

They are absolutely ideal in that respect. They are like mathematicians in disguise and they do mathematics in an extremely original manner which mathematicians would not be able to do. And they know a lot of physics. They are experts in physics but they are all in the same boat and that's

the problem. Supersymmetry assumes a very strong compatibility with complex numbers. Of course it simplifies things because complex numbers are infinitely simpler than real numbers. It was Andre Weil who said “complex analysis is beautiful, real analysis is dirty”. One can wish that physics is like that, why not. It is a beautiful dream but it is too early to believe that this is the truth. Because it is based on too many assumptions which make the mathematical theory simpler but do not accept what is handed to us by physics. So my attitude is different. I prefer to start from what physics gives us and try to understand it, find totally unexpected mathematical structure, like this cosmic galois group, in the already tested part of physics. At least one can be sure that this has something to do with nature.

## **MK**

What kind of NCG you suggest to physicists? Because there are parts that are totally neglected by them that would give new suggestions and new angles.

## **C**

Sure! For instance when you look at anomalies in quantum field theories on noncommutative spaces you find that the relevant cohomology is cyclic cohomology and the formulas become much more involved. So to treat anomalies it is probably a good idea to learn the part of NCG that deals with cyclic cohomology and the local index formula.

At the beginning when Witten wrote his first paper on the action for open strings he was using cyclic cohomology to define Chern-Simon action. In fact he was constructing a cyclic three cocycle on the convolution algebra of open strings. This was back in 86 and it did not last long.

Physicists tend to shift often and work on the last fad. I cannot complain

because at some point around 98 that fad was NCG after my paper with Douglas and Schwarz. But after a while when I saw Michael Douglas and asked him if he had thought more about these problems the answer was no because it was no longer the last fad and he wanted to work on something else. In mathematics one sometimes works for several years on a problem but these young physicists have a very different type of working habit. The unit of time in mathematics is about 10 years. A paper in mathematics which is 10 years old is still a recent paper. In physics it is 3 months. So I find it very difficult to cope with constant zapping.

## **GBK**

There is a problem here. You said 10 years. Sometimes physicists they have a database when they publish a paper tomorrow it will get ten citations but we are arguing here that in mathematics it takes 4 or 5 years before it comes to public attention.

## **C**

Of course, Of course.

## **GBK**

Most physicist don't understand this kind of measure

## **C**

The citation number when you look for instance at my own papers the one which is the most cited is the one that I wrote with Douglas and Schwartz.

**GBK**

Do you know how many citations have you got in that paper?

**C**

Almost a thousand I think, but it does not mean much because the main point of this paper was to relate an equation of string compactifications with the classification of holomorphic bundles on the noncommutative torus which we had done with Marc Rieffel in the early eighties. That older paper has very few quotations and the results there have been rediscovered in disguised form several times since then! I think the number of quotations is a very very strange measure. If you look at citations in mathematics then it doesn't make sense because there are very hard papers which very few people have read, and in fact there is really an inverse correlation between the difficulty of a paper and the number of people who actually read it, let alone quote it.

**GBK**

I was going to ask you a question but Masoud told me not to but still I will ask it in disguise. I was going to ask you how much do you think NCG has established itself as a very solid branch of Mathematics?

**C**

Now after 25 years there are many parts of NCG that have made contact with other fields of mathematics such as algebra, analysis and geometry. For instance cyclic cohomology has even made contact with topology through the Novikov conjecture. The links with physics have been there from the start and in essence the whole theory is translating the impact of the quantum revolution. The actual frontier is number theory and it will be hard to cross.

Still it is very striking that a noncommutative space as natural as the space of commensurability classes of  $\mathbb{Q}$ -lattices gives a spectral realization of zeros of the Riemann zeta function. So my impression from inside is to feel very happy with the development of the theory. For instance yesterday we had this problem session where one could write 21 examples of NC spaces and each was in fact a family of examples. So it's completely clear that it is providing new tools and spaces and creates lots of things to do, new territories to explore. If you look at it from the stand point of conservative mathematics where you have a standard list of topics like you have probability, algebra, geometry and all that, well it's not yet one of these topics. For the last international congress Yuri Manin tried to create a new section in ICM on NCG.

**GBK**

Math reviews?

**C**

No, not for math reviews. In math reviews it is already a section, he was trying to do that for the ICM but there is a lot of resistance and in fact mostly from nearby subjects. At the sociological level NCG is pretty well represented in Europe and it begins to exist in many other places including India, Australia etc... In the US there are very strong poles in localized places like Berkeley, Columbus, Penn-State, Vanderbilt etc but we still don't have representatives in the top universities.

**GBK**

You mean no one at Harvard and Princeton?

**C**

Yes, for instance.

**GBK**

Are you optimistic about the future of NCG?

**C**

I don't want things to happen artificially and I think the subject should exist only on its own value and for no other reason like sociology, fads etc... There is a lot of health in mathematics and this resistance before a new field gets accepted plays a positive role as a kind of filter. I much prefer that we have this resistance and that the only way to break it is to work more because that's a great incentive.

**GBK**

Yesterday they were talking in the cafeteria (of IPM) and I just eavesdropped and they were saying that the central figure is Alain and he is the prophet, the agitator and everything.

**C**

It is flattering but I don't think it is a good thing. In fact we are all human beings and it is a wrong idea to put a blind trust in a single person and believe in that person whatever happens. To give you an example I can tell you a story that happened to me.

I went to Chicago in 1996, and gave a talk in the physics department. A well known physicist was there and he left the room before the talk was



over. I didn't meet this physicist for two years and then, two years later, I gave the same talk in the Dirac Forum in Rutherford laboratory near Oxford. This time the same physicist was attending, looking very open and convinced and when he gave his talk later he mentioned my talk quite positively. This was quite amazing because it was the same talk and I had not forgotten his previous reaction. So on the way back to Oxford, I was sitting next to him in the bus, and asked him openly how can it be that you attended the same talk in Chicago and you left before the end and now you really liked it. The guy was not a beginner and was in his forties, his answer was "Witten was seen reading your book in the library in Princeton"! So I don't want to play that role of a prophet preventing people from thinking on their own and ruling the subject, ranking people and all that. I care a lot for ideas and about NCG because I love it as a branch of mathematics but I don't want my name to be associated with it as a prophet.

## **GBK**

But it is!

## **C**

Well, the point is that what matters are the ideas and they belong to nobody. To declare that some persons are on top of the ladder and can judge and rank the others is just nonsense mostly produced by the sociology (in fact by the system of recommendation letters). I don't want that to be true in NCG. I want freedom, I welcome heretics.

## **MK**

There is this phenomenon in mathematics where the old generation when they face new ideas in mathematics they resist and it's very hard for them to grasp it but for the next generation it's not a problem at all and they adopt it and they just swim through. Now NCG is a field that involves a lot of different things and it is really a hybrid. Now I have the feeling that for the next generation it will be much easier because they are seeing these things happening and they talk to people etc. So I'm sort of optimistic about the social situation.

## **C**

Exactly! The real purpose is to convince the young generation that there are lots of things to do in NCG, it is like a huge building site and we need a lot of help. So we don't need conferences but schools and this is exactly the motivation for being here: to create a school to convince the young people. We shall not change things by convincing the older generation. We care about convincing the younger generation.

## **GBK**

It's hard to convince the old generation.

## **C**

We don't care. Of course it's always useful to have criticism. On the other hand it's not the main point.

**GBK**

About two or three years ago the Clay institute announced 7 outstanding problems. Do you have other problems to add to that list?

**C**

It was in 2000. The main motivation of the Clay Institute was to attract public attention on mathematics and in that respect it worked perfectly. The wave went to extremely remote places and journals in most countries of the world talked about these problems. But one possible drawback of giving money for problems is to favor selfish attitudes. For instance if you are close to the solution and you don't want to share your ideas with anyone.

**MK**

Do you agree for example that Hilbert was not prophetic in his choice of problems and the course of mathematics in the 20th century was not predicted in Hilbert's questions?

**C**

Some of them played a role, but mathematics does not work like this. Nobody works on problems because they are well-known but mostly because they are interesting and relevant. The goal of the Millenium problems was to attract public attention on mathematics and in that respect it worked wonderfully.

**GBK**

Which one of those 7 problems is the most outstanding problem?

**C**

There is always an element of arbitrariness in choosing such problems. There are some problems that everybody agrees upon like the Riemann hypotheses. But Navier-Stokes? It is a typical nonlinear equation about which we would love to understand a lot more, but it's very hard to say that deciding for that equation between existence of smooth solutions or breakdown, which is a very difficult mathematical problem in analysis, will be truly relevant for the concrete instances where the equation applies. There is some element of arbitrariness. So nobody can be sure that these are the most important problems and it is very difficult for some of them even to formulate the question, while it would be clear to everybody what a breakthrough would mean. This is clear for the Yang-Mills problem for instance whose "mathematical" formulation is quite difficult.

**MK**

May I ask you a question about your trip to Iran and visiting the IPM? What was your feelings before coming here and how it changed, or if it changed at all, and what is your current impression of mathematics at IPM?

**C**

The thing that strikes me most is the number of talented students with a lot of freedom of thoughts. I didn't expect to see that. Of course I was also really impressed by the string theory group. I would say they are extremely creative and open minded and the way they are working is very impressive. I really admire this group!

**GBK**

Although their field is not very promising!

**C**

Well, as a group they are a wonderful group of people and they know physics very well. My personal opinion on the future of string theory in physics is irrelevant here. I'm an outsider. I'm not an insider. What I am sure of is that they are a blessing for mathematics.

**GBK**

And your impression about the IPM and the atmosphere here and the way the conference was run?

**C**

Well, my impression was extremely positive. The hope is that we did convince some of the young people that NCG is a proper subject in which they can do a lot of original work.

**GBK**

We are happy to have the good friends Masoud and Matilde here and we are happy to invest in many branches of mathematics but money is not enough. We can not do it just by that but maybe Masoud can originate Something here.

## C

Masoud is a key element. He convinced us to come and his role is crucial to develop NCG in Iran. He can also be the link between you and physics which is extremely important. I went to teach in China in April. Locally one can create a little wave but unless there is a way to establish more permanent contacts it's very difficult to have a lasting impact. It is crucial to reach a critical number of students and postdocs and it seems to me that now it is possible to create a kind of regional group of NCG. There are people in India, there are people in Beirut and they can begin to be independent with different taste and different approaches, like people in other places.

## GBK

We can make links with our physics department as well.

## C

Of course! String theorists know a lot about NCG but they have a different approach, they are our best allies and we have everything to gain in getting closer to them!

## GBK

They are very excited about this meeting too! This is not the first meeting we have had here for 16 years, but we have never done anything in common with physics. So this one was very successful and you came here and it was very impressive.

**C**

That's a great opportunity to develop NCG here. To collaborate with these bright string theorists! Perhaps they can give classes of physics to mathematicians and all that. You know one should be completely open to them.

**GBK**

So it is a blessing for this institute that they are here next to us.

**C**

It is wonderful!

**GBK**

One cultural question. Somebody told me that you got an offer from Harvard in 70's but you refused.

**C**

It was not in the 70's it was in the 80's.

**GBK**

You prefer the European approach to mathematics.

**C**

Of course. You know if I had been in the US I would have been obliged to enter into a system which I don't like at all. But it was not for this reason that I refused to go. I had accepted a position in College de France 6 months earlier and of course I was not going to move to another place after that.

**GBK**

But you prefer the European system.

**C**

Of Course.

**GBK**

They say that European system is very good for heroes but it's not for little guys for ordinary mathematicians.

**C**

In France we have a marvel which is the CNRS. It's a place where gifted people can get positions that they can keep for the rest of their lives. The main point is that it makes it possible for people like Lafforgue to think for many years about a problem without having to produce  $n$  papers per year and apply for an NSF grant. Young people can invest in long term projects which they could never do in a system with a short time unit.

**GBK**

This may work for some people and may backlash for others because they go there and do nothing for years.

**C**

You cannot decide before hand whom will be a Lafforgue and you will almost automatically have other people that will produce very little. It's a rule. It is the price to pay to eliminate this pressure to write  $n$  papers per



year which is nonsense in subjects which are really difficult. It takes 5-6 years to learn such a subject and you don't produce anything in that long interval. The French system is extremely efficient in that sense that it gives to some people the ability to work without being constantly bugged by the need to produce a paper. It is totally different from other systems but it is successful. Most of the CNRS researchers in mathematics are very interesting and productive mathematicians. The only problem is that there is not enough communication with universities and I've been trying to change that for many years. There is not enough flexibility to exchange between CNRS and the universities.

**GBK**

But in CNRS labs are located inside universities.

**C**

Yes their labs are in universities. It's really tough for university teachers who are next to them but are overloaded with teaching duties. And there is no easy way so far to exchange duties between the two groups : those who are at the university and those who are members of CNRS. I was teaching in the university for 6 years.

**GBK**

Which university?

**C**

Paris. It is very difficult to do research while one has teaching duties at university level. My impression at that point was that the available time

I had to do research, which used to be a connected interval had suddenly transformed into a Cantor set, I was constantly interrupted.

**GBK**

Where do you get your grants? from CNRS or...

**C**

First I was in CNRS for several years then I went to Canada. When I was in Canada I was offered a university position in Paris and I accepted it which was a big mistake. Then immediately after I started teaching I realized I had been stupid to leave and applied again for CNRS. It took me six years to be readmitted. This was between 75 and 81.

**GBK**

Before the Fields Medal?

**C**

Well, a little before before the Medal I finally got a CNRS position.

**GBK**

But you are now in college de France. Do you still get your research money from CNRS?

**C**

No, No, there is no research money like the NSF grants people get in the US for the summer research. France doesn't function with money.

**GBK**

You are hired by a certain institution,

**C**

Yes for instance CNRS,

**GBK**

So what about the money to do research, to travel to do these things? All these things come from CNRS?

**C**

There is very little money available to travel and a lot of bureaucracy to get that little amount from CNRS.

**GBK**

Your salary is paid by the CNRS.

**C**

I am at College de France now and get my salary from there.

**GBK**

And that is not fixed you get promotions.

**C**

No, it is fixed

## **GBK**

No increases? no raise?

## **C**

There is a maximum which one quickly attains. If you want the French system is not based on money but it might change. Intellectuals have for long cultivated a profound despise for money which at least was very present in my generation. When for example I applied to CNRS I applied for a low rank position because I cared so much more for “time” than for money.

## **MK**

What you are saying is very relevant to here because here in Iran they are trying to build research institutes and grant systems and it is important to take note of different systems that are available in the world and choose the one that is more appropriate.

## **C**

I believe that the most successful systems so far were these big institutes in the Soviet union, like the Landau institute, the Steklov institute, etc. Money did not play any role there, the job was just to talk about science. It is a dream to gather many young people in an institute and make sure that their basic activity is to talk about science without getting corrupted by thinking about buying a car, getting more money, having a plan for career etc.... Of course in the former Soviet Union there were no such things as cars to buy etc so the problem did not arise. In fact CNRS comes quite close to that dream too, provided one avoids all interference from our society which nowadays unfortunately tends to become more and more money oriented.

**GBK**

How many professors in mathematics are in the College de France?

**C**

4

**GBK**

You and Serre and...

**C**

No, Serre has retired. It is Zagier, Yoccoz, Lions and me.

**GBK**

Do you have teaching duties?

**C**

Yes I have to teach 18 hours per year on original stuff produced during that year.

**GBK**

In your own subject.

**C**

Of course, the idea is to show research while it is being done. The optimal case is discovering new things as you progress in the lecturing. I was able to

do that only 4-5 times in twenty years but indeed it happened. The idea for an optimal class is to explain some basic ideas as you start the class and to develop them as the lectures progress.

**GBK**

18 hours a year, and is it distributed according to a program or is it free?

**C**

All mathematicians lecture two hours a week for a period of three months.

**GBK**

And then you are free?

**C**

Your are completely free of course.

**GBK**

It's a good job!

**C**

In many ways it is a perfect homeopathic dose of teaching. Teaching is extremely useful for several reasons. The first is that you learn to give a talk, the second is that you are forced to check things carefully. It is not an expository talk, it has to be done with all the details. There are other reasons like getting fruitful interactions with the audience. Finally there is

no way one can become lazy since it is very demanding over the years to produce each year enough material for 18 hours of original work.

**GBK**

Are these talks are well attended?

**C**

Yes about 70 people attend.

**GBK**

You were criticizing the US way of doing research and approach to science but they have been very successful too, right? You have to work hard to get tenure, and research grants. Their system is very unified in the sense they have very few institutes like Institute for Advanced Studies but otherwise the system is modeled after universities. So you become first an assistant professor and so on. You are always worried about your raise but in spite of all these hazards the system is working.

**C**

I don't really agree. The system does not function as a closed system. The US are successful mostly because they import very bright scientists from abroad. For instance they have imported all of the Russian mathematicians at some point.

**GBK**

But the system is big enough to accommodate all these people this is also a good point.

## C

If the Soviet Union had not collapsed there would still be a great school of mathematics there with no pressure for money, no grants and they would be more successful than the US. In some sense once they migrated in the US they survived and did very well but I believed they would have bloomed better if not transplanted. By doing well they give the appearance that the US system is very successful but it is not on it's own by any means. The constant pressure for producing reduces the "time unit" of most young people there. Beginners have little choice but to find an adviser that is sociologically well implanted (so that at a later stage he or she will be able to write the relevant recommendation letters and get a position for the student) and then write a technical thesis showing that they have good muscles, and all this in a limited amount of time which prevents them from learning stuff that requires several years of hard work. We badly need good technicians, of course, but it is only a fraction of what generates progress in research. It reminds me of an anecdote about Andre Weil who at some point had some problems with elliptic operators so he invited a great expert in the field and he gave him the problem. The expert sat at the kitchen table and solved the problem after several hours. To thank him, Andre Weil said "when I have a problem with electricity I call an electrician, when I have a problem with ellipticity I use an elliptician".

From my point of view the actual system in the US really discourages people who are truly original thinkers, which often goes with a slow maturation at the technical level. Also the way the young people get their position on the market creates "feudalities" namely a few fields well implanted in key universities which reproduce themselves leaving no room for new fields.



## **GBK**

In the US there are so many mathematicians. Their system produces about 1200 new PhD's a year.

## **C**

And they can't find a position unless they belong to a field with the stamp of approval.

## **GBK**

This is massive! astronomical!

## **C**

But the problem is that whether or not they will find a position depends on whom will write their recommendation letters. I am not saying what kind of letter they will get since all these letters look alike in their emphatic style. The result is that there are very few subjects which are emphasized and keep producing students and of course this does not create the right conditions for new fields to emerge. At least in France, if you have a position in CNRS you are allowed to do whatever you want and people are given the maximum freedom of thinking without any unhealthy social pressure to work in this or that field if one wants to secure one's future!

## **GBK**

But here in Iran our old system was modelled after the French system. In the University of Tehran long time ago most of the professors were educated in France, some before world war II and some after, but the system was not at all efficient. Nobody did anything.

**MK**

but they didn't do research for other reasons, like social and political instability.

**C**

You didn't have CNRS. What is vital for research is CNRS.

**MK**

These people were the first generation of Iranians who had contact with modern mathematics and it was very hard to continue that in Iran back then.

**GBK**

If you adopt something like CNRS with no control no checking then nobody would do research.

**MK**

But this is not clear.

**GBK**

France had the tradition of science for 500 years. You go to Sorbone and you see all these big names. The CNRS system works there because there are some weighty bones in CNRS.

**C**

Just creating an analogue of CNRS wont resolve the problem of course but clearly that type of system also succeeded in the Soviet Union; the system was similar to CNRS with large numbers of researchers and they were doing great things.

**GBK**

This is probably two different kinds of systems. France and Russia are different from Iran.

**MK**

I'm not pessimistic because pursuit of abstract ideas and higher knowledge has strong roots in persian culture. Science and knowledge just for the sake of science and knowledge.

**C**

Exactly.

**MK**

Of course there will be same cases of people who will abuse the system and won't produce anything.

**C**

This you can not avoid anyway, it is a statistical law and if you try to remove the tail of the curve you wont succeed, you'll just shift it.

## **MK**

But the average result will be very good.

## **GBK**

But if the numbers are very big and if the numbers are small the system will suffer. For instance here at IPM if we don't check anybody's outcome- we give this money and ask people to do research-and if we don't check them and there is no kicking out we will lose the efficiency

## **MK**

But only very good people are hired in CNRS.

## **C**

Yes the thing is that CNRS is extremely difficult to get into. It's extremely competitive. But once you made it you can stay for the rest of your life and no real evaluation is performed which has obvious negative aspects. A system which would be slightly better than the actual one would be the following : first admit a large number of young people in CNRS for 6 years. Next, after 6 years they all would have to leave CNRS and teach in the university at various levels. Then, and only then, they would be able to apply again to CNRS. They would reenter the CNRS and be given a permanent position only in this second stage where obviously the competition would be fierce. If they would not succeed in the second stage they would just stay at the university where they teach. This could improve the system in putting more emphasis on freedom for younger people and creating a second filter to diminish the number of people who stay in CNRS and don't produce anything. They would be in universities and teach, which is fine.

## **GBK**

If you do alright in the first 6 years you go to universities and then reapply if you do moderately well.

## **C**

After six years you imperatively would have to go to university and only then you would be able to apply again to CNRS.

## **GBK**

But if you do very good in both stages then you can stay in CNRS for the rest of your life.

## **C**

If you did extremely well you would be able to reenter the CNRS where then would become a director of research and would have a lot of students and more freedom to think than university teachers.

## **GBK**

So there are two stages of testing.

## **C**

It's not what is done now. Now if you enter the CNRS you can stay there for the rest of your life which is not so good because it prevents more young people to get in. We need many more open doors between the CNRS and the university.

**MK**

How do you choose your research problems? You seem to go back to problems you studied once and look at them with new tools that you discover.

**C**

Sure I never abandon problems. On problems that I care for I will be persistent. I think in mathematics it is extremely important to be persistent. The point is not being brighter or faster. Forget it! What is important is to never abandon a problem.

**MK**

Do you have mathematical heroes?

**GBK**

He is coming from Canada. He looks for heroes!

**C**

The impression that I have after many years is that each human being is unique and could well be a hero of some kind depending on the circumstances. In mathematics the problem is not really to be a hero but to be able to be patient and to apply enough intensity in one's research.

**MK**

But also as a teacher I think it's important to give role models to young people, something young people can aspire to be, to emulate and look up to.

**C**

I'm the worst person to be a role model.

**MK**

You are a very hard role model to try to emulate of course!

**GBK**

So you don't have any heroes but who do you admire most?

**C**

Life forced certain mathematicians to be heroes and of course Galois is a marvelous example in that respect. He spent most of the last year of his short life in jail and he was forced to spend his days among a noisy crowd of bandits that at some point forced him to drink a bottle of liquor making him awfully sick. He had this incredible strength of character and was able to keep working in such an environment. He was barely twenty years old and looked as if he had been already fifty... but he produced these marvelous ideas which still propagate in the minds of mathematicians.. despite the harshness of his life he was able to keep producing seminal ideas. Most mathematicians are not heroes for sure.

**GBK**

Do you have a good memory of Ecole Normale?

**C**

Sure! I can tell you what happened when I entered Ecole Normale in 66. I was coming from Marseille and had undergone two years of preparatory school which was “bourrage de cranes”. We were learning how to calculate integrals, drawing graphs of functions etc.. and I was fed up with it. When I arrived in Ecole Normal essentially I took one year off. It was like a hotel in Paris and we had fun, except that we were discussing mathematics all the time with the other students. After that year I started working on my own research.

**GBK**

You didn't have to take courses?

**C**

I didn't go to any classes and didn't know where the university was, so when I had to take the exam my friend had to take me to the exam room and I saw the university for the first time!

**GBK**

So it was a leisure time!

**C**

No, it was not leisure it was freedom, it was some kind of reaction against the preparatory schools in which we were taught recipes to pass exams. I just wanted to think quietly by myself and enjoy life of course, and that was given to us in Ecole Normale.



**GBK**

it was before 1969?

**C**

I entered in the fall of 66 and then came the events of 1968.

**GBK**

That was a turbulent time.

**C**

Yes 68 was a turbulent time. We had already built the right kind of mood for 68.

**GBK**

So you were in Paris in the best place and in the best time.

**C**

Yes it was a good time. I think it was ideal that we were a small group of people and our only motivation was pure thought and no talking about careers. We couldn't care the less and our main occupation was just discussing mathematics and challenging each other with problems. I don't mean "puzzles" but problems which required a lot of thought, time or speed was not a factor, we just had all the time we needed. If you could give that to gifted young people it would be perfect.

**GBK**

For how many years were you there?

**C**

For 4 years, but as I said the first year was a free year and then I had to pass aggregation and I refused. I was one of the two people who refused to undergo that exam because I didn't want to go back to school time since I had barely managed to survive that before.

**MK**

How did you come across your first research problem? You mentioned that you had a clear idea what you wanted to do.

**C**

I was working on the location of roots of polynomials. So you are given a polynomial and you want to know where the roots are in the complex plane. That was my first problem. I had found a notion, some kind of weak ordering in the complex numbers, which was simplifying all the proofs of the theorems in the books I was looking at and one could go a bit further. I worked on that for some time.

**MK**

Then you moved to Von Neumann algebras quickly.

## C

It took me a lot of time to find what I really wanted to do. When I moved to von Neumann algebras I had the impression that it was a part of mathematics that was widely accepted. It was like moving from a very small little village to a big town and each time I thought I was already in the center of mathematics. But then when I arrived in IHES in 76, I realized that it was not really the case. But I never had the wish to move to main stream. I really hated the arrogance of some people. My only desire was to do things that were orthogonal to what these guys were doing. This implied being as far from algebraic geometry as one could be.

## GBK

Did you ever go to the IHES during the Grothendieck's time?

## C

Not at that time mostly because from the distance it looked like such an arrogant group. After all these years I finally took the time to read Grothendieck's book "recoltes et semailles" and this allowed me to really understand his personality much better. I realized that behind this arrogant appearance there lies the most admirable human being and I regretted not to have had the chance to talk to him. In fact the correspondence between Grothendieck and Serre is available in print and it is a marvelous testimony of their power and personalities. What is most striking is this great intellectual honesty, a model if any!

## GBK

Did you ever join Bourbaki?

**C**

Well I did for one year but I got quickly discouraged. They had already done the point somehow. I found out that they had about 500 manuscripts each of around 200 pages which were sitting in the cupboard somewhere, some already for twenty years. So writing one more of these looked like such a loss of energy. When they had someone like Dieudonne who wrote during his mathematical life something like 80000 pages of mathematics in a relentless way, the machine would function. But when I arrived, he was already gone and there was nobody to replace him. So it became more like a club and I didn't care to participate

**GBK**

When was this?

**C**

It was back in 78 or something like that.

**GBK**

Then you announce that your were leaving?

**C**

You don't have to. You just don't go to the meetings, you just stop going. Another reason to leave was that they had a life style which was unpleasant. People would leave without saying good bye, being rude was the main feature of the founders they seemed to cherish. I found it very irritating. Clearly the founders did great things..

## **GBK**

For sometime.

## **C**

For some time. Their integration book is terrible but they produced these beautiful books in Algebra, and the whole series on Lie groups which is wonderful.