3. Momentum Principle in Open Channel flows
The momentum equation commonly used in most of the open channel flow problems is a linear-momentum equation. This equation states that the algebraic sum of all external forces acting in a given direction on a fluid mass equals the time rate of change of linear-momentum of the fluid mass in that direction. In a steady flow the rate of change of momentum in a given direction will be equal to the net flux of momentum in that direction.

![Figure 1.1 definition sketch for the momentum equation](image)

The figure shows a control volume (a volume fixed in space) bounded by section 1 and 2, the boundary and a surface laying above the free surface. The various forces acting on the control volume in the longitudinal direction are:

a) Pressure force acting on the control surfaces, $F_1$ and $F_2$.
b) Tangential force on the bed $F_3$,
c) Body force, i.e the component of the weight of the fluid in the longitudinal direction $F_4$.
d) $F_a$ is the air resistance at the free surface.

\[ \sum F_x = W \sin \theta + F_1 - F_2 - F_3 - F_a = M_2 - M_1 \]

Here $F_1$ and $F_2$ are hydrostatic pressures on the section 1 and 2, $W$ is the weight of the control volume, $\theta$ is the slope of the bed with the horizontal, $F_3$ is the boundary friction over the length $\Delta X$ and $F_a$ is the air resistance at the free surface. Generally speaking $F_a$ is negligible and it is customary to neglect $F_3$ also when $\Delta X$ is small. In which $M_1$ and $M_2$ are momentum flux entering and leaving the control volume $\beta_1 \rho Q v_1$ and $\beta_2 \rho Q v_2$.

\[ \sum F_x = W \sin \theta + F_1 - F_2 - F_3 - F_a = Q \rho (\beta_2 v_2 - v_1) \beta_1 \]

In practical applications of the momentum equation, the proper identification of the control volume and the various forces acting on it are very important. The momentum equation is practically useful tool in analysing rapidly varied flow (RVF) situation where energy loss are complex and cannot be easily estimated.

With assumption that $\theta=0$ and $\beta_1=\beta_2 = 1$ and very small tangential force the above equation becomes:

\[ \gamma Z_1 A_1 - \gamma Z_2 A_2 = \rho Q(v_2 - v_1) \]

where $Z_1$ and $Z_2$ are the distance to the centroid of respective cross sectional flow areas $A_1$ and $A_2$. 

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\[ y_1A_1 - y_2A_2 = \frac{Q^2}{g} \left( \frac{1}{A_2} - \frac{1}{A_1} \right) \]

\[ \frac{Q_1}{gA_1} + Z_1A_1 = \frac{Q_2}{gA_2} + Z_2A_2 \] is known as specific momentum of force function.

Hydraulic jump

Hydraulic jump is formed when ever supercritical flow changes to subcritical flow, at the jump location there is a sharp increase in water surfaces and a considerable amount of energy is dissipated due to turbulence. At present we are interested in developing a relationship between the flow depth and flow velocities upstream and downstream of the jump of the flow. Upstream and downstream depths of the flow are called sequent depths or conjugate depths.

To simplify the derivation: we will consider a rectangular horizontal channel, since the amount of energy losses in the jump is not known in advance we cant apply the energy equation directly. However since the length of the jump is very short, the losses due to the shear at the channel bottom and sides are small as compared to the pressure force and may be neglected. In addition since the channel is horizontal, the component of the weight of water in the flow downstream direction is zero.

\[ F_1 - F_2 = Q \rho (\beta_2 v_2 - \nu_1 \beta_1) \quad \text{momentum correction factor } = 1 \]

\[ \frac{1}{2} \rho g y_1^2 - \frac{1}{2} \rho g y_2^2 = Q \rho (v_2 - v_1) \]

\[ Q = A_1 v_1 = A_2 v_2 \]

consider a unit width rectangular channel

\[ \frac{1}{2} g (y_1^2 - y_2^2) = q(v_2 - v_1) \]

\[ \frac{1}{2} g (y_1 - y_2)(y_1 + y_2) = q(y_2 - y_1) \]

\[ y_1 y_2(y_1 + y_2) = \frac{2q^2}{g} \]

\[ y_1 y_2(y_1 + y_2) = \frac{2v_1^2 y_1^2}{g} \quad \text{for rectangular section } F_1 = \frac{y_1}{\sqrt{g v_1}} \quad F_1^2 = \frac{y_1^2}{g v_1} \]

\[ \frac{y_2^2}{y_1^2} (y_1 + y_2) = 2F_1^2 \quad \text{solving for } y_2/y_1 \]

\[ \frac{y_2}{y_1} = \frac{1}{2} \left( -1 + \sqrt{1 + 8F_1^2} \right) \]
Problems

1. A hydraulic jump assisted by a two dimensional block is formed on a horizontal apron as shown in figure below. Estimate the force $F_D$ in KN/m width on the block when a discharge of 6.64$m^3/s$ per m width enters the apron at depth of 0.5m and leaves it at a depth of 3.6m.

2. Show that the energy lost in the jump of a horizontal, frictionless rectangular channel is:

$$E_L = \frac{(y_2 - y_1)^3}{4y_1y_2}$$

3. A hydraulic jump occurs in a horizontal rectangular channel with sequent depths of 0.7m and 4.2 m. Calculate the rate of the flow per unit width, energy loss and the initial Froude number.

4. A hydraulic jump occurs in a horizontal rectangular channel at an initial Froude number of 10.0. What percentage of initial energy is lost in this jump?

5. A hydraulic jump in a rectangular channel has a Froude number at the beginning of the jump $F_1=5$. Find the Froude number $F_2$ at the end of the jump.