

### Roots of unity

Write down the seventh roots of unity. Hence, or otherwise, find all the roots of the equation  $z^7 = 8(1-i)$ , giving each root in the form  $re^{i\theta}$ .

Deduce that if  $\left[ \frac{w-1}{\sqrt{2}(w+1)} \right]^7 = 8(1-i)$ , then  $\operatorname{Im}(w) = \frac{4 \sin \alpha}{5-4 \cos \alpha}$  where  $\alpha = \left( \frac{8k-1}{28} \right) \pi$  and  $k = \pm 3, \pm 2, \pm 1, 0$ .