- Hi,

  Now that you have a comprehensive checklist to know what to focus on when you revise, it is also important to know something about effective problem-solving.

  I will describe Polya's problem-solving approach and illustrate its application with an example. The approach consists of four steps with some further points under each step.

  Step 1: Understand the problem

  What am I given?
  What do I need to solve?

  Step 2: Devise or make a plan

  What approaches are available for me to solve?
  Have I solved similar problems before?
  Are there smaller problems for me to solve?

  Step 3: Carry out the plan

  I will write out the steps of my solution.

  Step 4: Look back at the completed solution

  Is my final answer a reasonable one?
  Are my steps clear, logical and error-proof?
  Can I check the answer with calculator?
  Can I substitute back?
  Can I obtain the same answer via another approach?

  Now let's look at an example involving differential equations:

  The curve C satisfies the equation y " = sin^3 x. It is given that C cuts the y-axis at -1 and passes through the point (4/pi, 2/9). Find y in terms of x.

  Step 1: Understand the problem

  What am I given?

  1. y " = sin^3 x

  2. Points (0, -1) and (4/pi, 2/9) lie on C.

  What do I need to solve?

  1. Find the particular solution, because I am given particular points.

  2. Express y in terms of x, because it is required by the question.

  Step 2: Devise or make a plan

  What approaches are available for me to solve?

They are direct integration, variable separable, substitution which I have been exposed to.

Have I solved similar problems before?

Perhaps, I may have seen it before in lectures, tutorials, tests, exams, etc.

Are there smaller problems for me to solve?

1. Yes, I need to find dy/dx before I can find y in terms of x.

2. The expression sin^3 x may need to be simplified by applying a suitable trigonometric identity before I can integrate it.

3. Since I am given two points that lie on C, I will need to find constants in the particular solution.

Step 3: Carry out the plan

I will write out the steps of the solution as follow.

y " = sin^3 x
= (sin x)(sin^2 x), by Pythagorean identity
= (sin x)(1 - cos^2 x)
= sin x - (sin x)(cos^2 x)

I recognise that -(sin x)(cos^2 x) looks like f '(x) . {f(x)}^n, so I can integrate the expression directly.

Integrating both sides wrt x,
y ' = -cos x + 1/3 cos^3 x + c, so I have solved a smaller problem.

y ' = -cos x + 1/3 (cos x)(1 - sin^2 x) + c, following the same approach earlier.
= -cos x + 1/3 cos x - 1/3 (cos x)(sin^2 x) + c
= -2/3 cos x - 1/3 (cos x)(sin^2 x) + c

Integrating both sides wrt x again,
y = -2/3 sin x - 1/3 (1/3 sin^3 x) + cx + d
= -2/3 sin x - 1/9 sin^3 x + cx + d, so I have obtained a general solution and another smaller problem has been tackled.

Since (0, -1) lies on C, d = -1.

Now y = -2/3 sin x - 1/9 sin^3 x + cx - 1.

Since (4/pi, 2/9) lies on C, -2/3 - 1/9 + (4/pi) c - 1 = 2/9
=> c = pi/2

Finally, the particular solution is y = -2/3 sin x - 1/9 sin^3 x + (pi/2) x - 1.

Step 4: Look back at the completed solution

Is my final answer a reasonable one?

It is, because the expression is trigonometric given that the original expression is trigonometric.

Are my steps clear, logical and error-proof?

Yes, though I must be careful with minus signs.

Can I substitute back?

Yes, I can check using the two points that lie on C.

Can I obtain the same answer via another approach?

No, but I can differentiate y twice and see if I could obtain the expression

y " = sin^3 x,

though it may be time-consuming in the exam. I may do it if it is a tutorial question or a question that I'm doing during my revision.

I hope this example convinces students that the Polya's method is effective, especially in its steps 1 & 2. By carrying out the first two steps rigorously, it will bring about the solution naturally.

Thanks for reading!

Cheers,
Wen Shih