

H2 Mathematics JC 1 Promotional Examination Practice Paper
Answer all questions in 3 hours

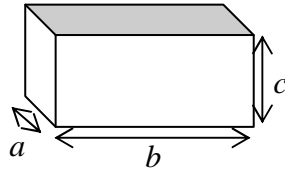
Knowledge from these topics will be necessary for this practice paper:

- Partial fractions
- Binomial expansion
- Inequalities & equations
- Arithmetic and geometric progressions
- Summation of series
- Mathematical induction
- Functions
- Curve sketching
- Transformation of curves
- Applications of differentiation
- Maclaurin's expansion
- Small angle approximations
- Integration techniques and its applications

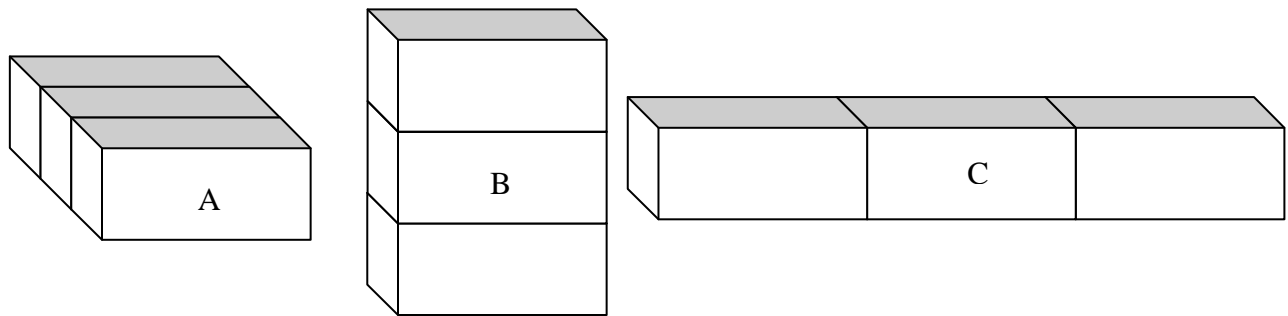
Good luck for your actual examination!

1 Given that θ is sufficiently small for θ^3 and higher powers of θ to be neglected, express $\frac{2 - \tan \theta}{4 + 2 \sin \theta}$ as a quadratic expression in θ . [4]

2 A small cuboid has dimensions $a \times b \times c$ units, as shown in the diagram.



Three such cuboids can be arranged differently to form 3 larger cuboids, A, B and C, as shown in the diagrams below. The sum of the edges of A, B and C are 76.4 units, 54 units and 107.6 units respectively.



By using a system of linear equations involving a , b and c , find the volume of a small cuboid. [5]

3 Solve the inequality $\frac{x+3}{(x+2)(x-1)} > 1$. [4]

Hence solve $\frac{3-x}{(2-x)(x+1)} < -1$. [2]

4 Given that $y = e^{\sin^{-1} 2x}$, show that $(1-4x^2) \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} = 4y$.

Find the Maclaurin's series for y , up to and including the term in x^3 . [6]

5 Show that $\sum_{k=1}^n \frac{1}{k(k+1)} = \frac{n}{n+1}$. [2]

By writing $\frac{5r+2}{r(r+1)(r+2)}$ in the form $\frac{p}{r(r+1)} + \frac{q}{(r+1)(r+2)}$, find, in terms of N , an

expression for $S_N = \sum_{r=1}^N \frac{5r+2}{r(r+1)(r+2)}$ and deduce the limit of S_N as $N \rightarrow \infty$. [4]

6 In an infinite series of concentric circles, the radius of the first circle is 12 cm, and the radius of each circle after the first is $\frac{2}{3}$ that of the previous circle.

(i) The radius of the n^{th} circle is less than 1 cm. Find the least value of n . [4]

(ii) Find the total area of this infinite series of concentric circles, leaving your answer in terms of π . [3]

7 Prove by induction that $\sum_{r=1}^n (r+1)2^r = n(2^{n+1})$. [4]

Hence find $\sum_{r=1}^n (2r+1)2^r$ in terms of n . [3]

8 The curve C has equation $y = \frac{6x}{x^2 + ax + 1}$, where a is a positive constant and $0 < a < 2$.

(i) Explain why C has no vertical asymptotes, and determine the coordinates of the stationary point(s) of C in terms of a , [5]

(ii) Sketch C . [2]

9 The functions f , g and h are defined as follows:

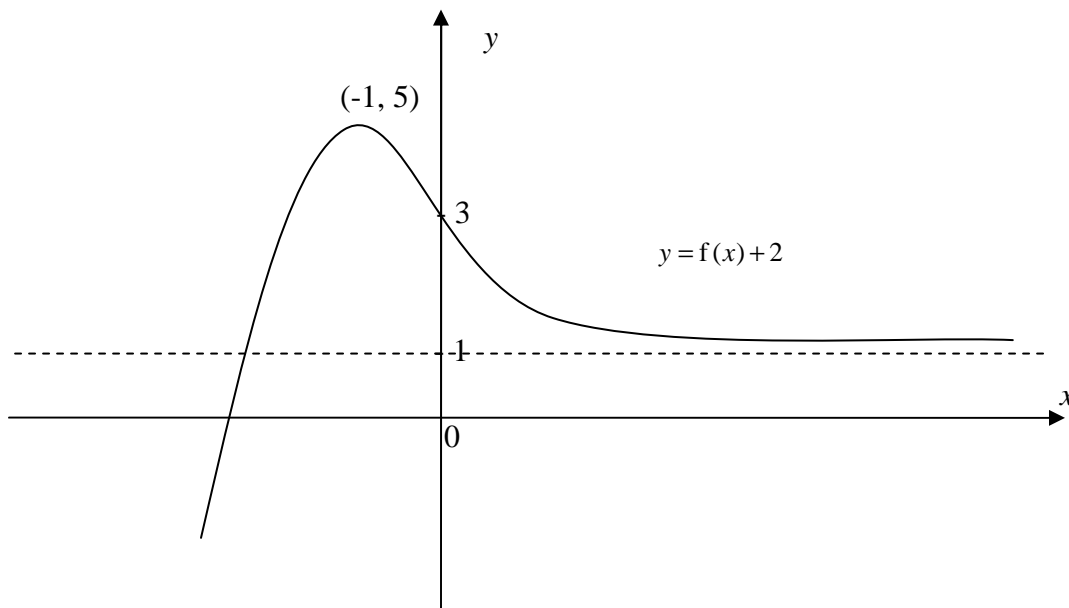
$$f : x \mapsto 4 - x^2, \quad x \in \mathbb{R}, x < 0,$$

$$g : x \mapsto e^{1-2x}, \quad x \in \mathbb{R}, x > 0,$$

$$h : x \mapsto \ln x, \quad x \in \mathbb{R}, x > 0.$$

- (i) Define f^{-1} in a similar form. [3]
 (ii) Sketch the graph of $y = hg(x)$ and find the range of this function. [2]
 (iii) Show that hf does not exist. Find the maximal domain of f for which hf exists. [3]

10 The graph below is $y = f(x) + 2$.

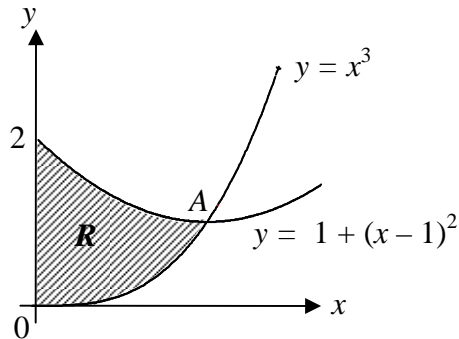


Sketch, on separate diagrams,

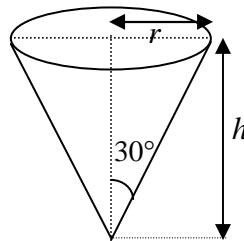
- (i) $y = -2f(x)$, [3]
 (ii) $y = f(|x|)$, [2]
 (iii) $y = f'(x)$. [3]

Indicate clearly any asymptotes, turning points and axial intercepts.

- 11 The diagram shows the region R in the first quadrant bounded by the curves $y = x^3$, $y = 1 + (x - 1)^2$ and the y -axis. The coordinates of the point of intersection, A , between the two curves are $(1, 1)$.



- (i) Find the exact area of R . [4]
- (ii) Find the volume generated when R is rotated through four right angles about the y -axis. [5]
- 12 Find
- (a) $\int x^2 \tan^{-1} x \, dx$, [4]
- (b) $\int \frac{x+2}{\sqrt{1-3x^2}} \, dx$. [4]
- (c) Use the substitution $x = \sin \theta$ to evaluate the integral $\int_0^1 x^2 \sqrt{1-x^2} \, dx$. [5]
- 13 (a) A conical paper cup, with semi-vertical angle 30° , is held with its axis vertical and its vertex downwards, as shown in the diagram. If the cone is filled with water which runs out at the rate of $3 \text{ cm}^3 \text{ s}^{-1}$, find the rate at which the level of water in the cone is falling when the depth is 4 cm, giving your answer in exact form. [4]



[The formula for the volume of a cone is $V = \frac{1}{3} \pi r^2 h$.]

- (b) A pyramid of vertical height h cm has a square horizontal base with side of length x cm and the vertex is directly above the centre of the base.

A cm² denotes the total surface area (i.e. the area of the base and the area of the four sloping faces). The volume, V cm³, of the pyramid is given by

$$V = \frac{1}{3}x^2h.$$

By expressing h^2 in terms of A and x , or otherwise, show that

$$V^2 = \frac{Ax^2(A-2x^2)}{36}. \quad [5]$$

Hence, for fixed A , find the maximum value of V^2 in terms of A and show that the corresponding value of $\frac{h}{x}$ is $\sqrt{2}$. [5]

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