## Alternatives to Parametric , Ordinary Least Square (OLS) Statistics

**Alternatives**

- **Rank (distribution-free) Statistics**
- **Permutation Tests**
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### Introduction

Note that the following is limited by my knowledge of the vast array of statistical procedures, statistical software and the even more vast array of literature. As an example of this vastness, the scale estimator called Gini's mean difference was published in statistics journal articles by five different authors during the first 75 years of the $20^{th}$ century. The last four did not know of Gini's article, nor did the reviewers or journal editors. Since that time, the literature has expanded enormously, although fortunately for Americans, most is in English.

Although frequently not taught in schools, there are several alternative statistics that are robust to alpha errors and have greater power (beta) than parametric statistics when faced with non-Gaussian distributions. Such alternatives fall into three general groups:

**Rank statistics** (distribution free), such as the Wilcoxon Rank Sum and Sign Rank and the Kruskall-Wallace)

Easy to understand,
Readily available in commercial statistical software,
Alternatives for many of the most commonly used OLS statistics,
Robust to alpha at the Gaussian, and
Sometimes considerably more powerful than OLS in the face of contamination.

### Permutation Tests

Although these have promise, and the literature is growing, their availability remains limited, and the "jury is still out" regarding their performance

### So-called Robust Statistics and Tests

Again, limited availability and although some are robust (like the Trimmed t with Winsorized variance), others exhibit problems in the face of specific situations, thus it is probably dangerous to generalize these across all situations.

Thus, rank tests, where available, appear the safest and are certainly the easiest choice.

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## Rank Test Statistic Summary[1]

Below is a quick overview of rank tests, what they do, and how to compute them using statistical packages (a very simple exercise in most cases).

Rank statistics reduce the information available from data by using only the order of observations, and not the distance between observations' values. Thus, the highest value in a distribution of say 50 salaries, whether $100.00 or $100,000,000.00 gets the rank of 1, and the lowest, whether $5.00 or $500,000.00 gets the rank of 50 (assuming no tied ranks). In the two-group independent means comparison case, the ***t***-test, we use the Wilcoxon Rank-Sum/Mann-Whitney U. Here ranks are computed across all observations in both groups, then the mean ranks for the different groups are used to compute the test statistic. In our salaries case, if one group was comprised of gravediggers and the other CEOs of major corporations, we might see a mean rank of 13 for the CEOs and 37 for the gravediggers. One major advantage of this approach over the ***t***-test comes from the reduced variance estimation. If, for example, the mean of the CEO group is $200,000.00, one can see that the difference between that and the highest ($100,000,000.00) squared (as it is under OLS statistics) is a huge number and considerably reduces the chance of having significant differences (here using unpooled variance estimates), whereas in the rank case, the difference between 1 and 13 squared considerably reduces the influence of extreme cases.

Rank tests are probably the easiest to understand, are robust to alpha at the Gaussian, and frequently exhibit large power advantages over parametric statistics in the face of certain forms of contamination. Additionally, following steps outlined by Conover & Iman (1981 ),[2] many software packages have the capacity to compute these.

## How to Compute Rank Tests using Packaged Statistical Software

Many packages contain specific procedures/subroutines to compute certain rank test statistics, such as the Spearman Rho (SAS – Proc Corr; SPSS – NPAR Tests ), the Wilcoxon Rank Sum/Mann-Whitney U (SAS – Proc Rank;[3] SPSS – NPAR Tests ) and the Kruskal-Wallis (SAS – NPARONEWAY; SPSS – NPAR Tests).

For those not having specific procedures/subroutines (below is a list of parametric tests for which this transformation is appropriate), the only action required is to first transform your dependent data to ranks (SAS – Proc Rank; SPSS – Rank Order)) and then run the appropriate parametric statistical tests on the ranked data. The ***p*** values you obtain are almost identical to those that would be obtained when using the rank tests themselves (see Conover & Iman, 1981 for regression procedures).

Below is a list of common statistical tests and their appropriate rank alternatives.

[1] The best and most clearly written text I have ever seen on the topic of Rank statistics is E.L. Lehmann (1975) *Nonparametrics: Statistical Methods Based on Ranks* (McGraw-Hill, NY). [2] Conover, W.J. and Iman, R.L. (1981). Rank transformations as a bridge between parametric and nonparametric statistics. *The American Statistician* , 35, pp. 124-129 [3] SAS Procedures Manual p. 498 – 'A two-sample *t*-test applied to the ranks is equivalent to a Wilcoxon rank sum test using the *t* approximation for the significance level'

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**Parametric Statistics for which rank tests are appropriate:**
Pearson r (Spearman rho)
Student's ***t*** (Wilcoxon Rank Sum/Mann Whitney U)
Paired-difference ***t*** (Wilcoxon Sign Rank)
Oneway ANOVA (Kruskall-Wallace)
Factorial ANOVA (for main effects ONLY) - effectively Kruskall-Wallace
Hottelling's T (with independent samples)
Multiple Regression (becomes ranked regression – parametric significance levels are close approximations to the rank levels)

**Parametric Statistics for which rank statistics are NOT appropriate:**
Interaction effects of Factorial ANOVA Repeated Measures (beyond the paired difference ***t*** – Wilcoxon Sign Rank) Factor Analysis Discriminant Analysis MANOVA

**Quality Control Statistics**
Certain Quality Control Statistics are non-parametric, and others could be computed using an alpha trimmed mean rather than the arithmetic mean as an estimate of center, and either a trimmed or Winsorized variance estimates for scale when setting boundaries. This would be particularly appropriate if it appears that the distributions assumed by the process control statistics differ from the obtained distributions (Note that DataDef can be of help here, although different statistics assume different distributions, perhaps the most common being the Gaussian, binomial and Poisson). Having said that, one must realize that the primary purpose of quality control techniques is to identify extreme cases (those that fall beyond tolerance limits), therefore, it may be more appropriate to apply the comparatively extreme-sensitive parametric-based statistics than statistics that reduce the influence of outliers and leverage points (like the alpha trimmed mean and Winsorized variance estimates). You may obtain more false positives using the mean and standard deviation, but in process control one usually wishes to take no chances that the process itself is out of control and to isolate and eliminate any assignable causes for lack of control. There are also, of course, two primary parts to the Quality Control process:
(1) process control – no assignable causes, and
(2) process capability – produces a large percentage of "useful" outputs (e.g. – The Boyer Commission report, 1998, suggests that the Higher Education process is neither in control nor capable).

**Survival Analysis**
Several rank tests are available for Survival Analysis, for example there is a parallel of the Wilcoxon rank sum when comparing two sets of scores. I am not very familiar with this area.

**MultiLevel Modeling (MLM)**
See the Multool.xls at the top and the explanation attached to that explanation.
MLM is a rather nice approach to conducting analyses with pretty much all of the capacities but considerably less restrictive assumptions that OLS Approaches.

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