<u>A Christian Perspective on Teaching Mathematics</u>

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My entry point for a Christian perspective on teaching mathematics is to offer

<u>A Brief Sketch of a Christian Worldview</u>

If I were to ask you what Jews believe you would probably know that the one verse answer is Deut 6:4, the Shema`, "Hear O Israel, the LORD our God, the LORD is one. And you shall love the LORD your God with all your heart and with all your soul and with all your might." However, not many Christians seem to know that there is a single verse in the New Testament, clearly related to the Shema, which gives a remarkable and succinct Christian confession concerning God the Father, the Lord Jesus Christ, ourselves and the universe. That verse is 1 Corinthians 8:6:

"Yet for us there is one God, the Father, from whom are all things, and for whom we exist, and one Lord, Jesus Christ, through whom are all things, and through whom we exist."

The relationship to Jesus Christ is spelled out more fully, especially concerning redemption, in the well-known verses Colossians 1:15-20:

"He is the image of the invisible God, the firstborn of all creation.
For by him all things were created, in heaven and on earth, visible and invisible,
Whether thrones or dominions or rulers or authorities--
All things were created through him and for him.
And he is before all things, and in him all things hold together.
And he is the head of the body, the church.
He is the beginning, the firstborn from the dead, that in everything he might be pre-eminent.
For in him all the fullness of God was pleased to dwell,
And through him to reconcile to himself all things, whether on earth or in heaven, making peace by the blood of his cross."

From this follows certain very important "vertical" and "horizontal" dimensions to our life in God's creation. First of all, we can say that loving God is to be central to all of our activities, including mathematics. Closely related to this central demand of love of God, the mathematical education that we are engaged in will be a failure if it does not make clear to our students that "all things" (i.e., everything in the created universe) are from God and will have their ultimate destination in Him; and that more specifically "all things" are <u>through</u> Jesus Christ, that they <u>cohere</u> in Him, and that the way in which they will come to their ultimate destination in God is through the cosmic reconciliation that Jesus Christ obtained by His death upon the cross.

I think it is helpful to examine in the light of these statements the dominant models of mathematical education that have been used in schools. At the risk of grossly oversimplifying things, I will put forward the provocative thesis that at least on the basis of what one can find in school mathematics texts, there have only been three <u>dominant</u> models of mathematical education in the western world, namely, the Platonist (from Plato right down to the Russians' launch of Sputnik 1; the formalist, epitomised by the New Math along with various reactions to it, up till about the 1990's; and various versions of constructivism from the late 1990's.

Platonism.

Though Greek mathematics is not the oldest example we have of mathematical paedagogy (the Rhind papyrus gives us a fascinating insight into the way that mathematics was taught to Egyptian youth, and clay tablets from Sumeria give us an insight into Babylonian mathematics), Greek mathematics has been disproportionately influential upon Western mathematical education. Two features stand out: the Greek emphasis upon logically developed mathematics, and, with the notable exception of Archimedes, their despising of applications. Their emphasis upon logical reasoning was closely connected with their search for certainty. Plato's ideal world of eternally existing perfect straight lines, triangles, squares and circles gave an independent self-sufficient existence to these objects, which existed independent of God. To see that Platonism is not a dead issue, let us look at the view of the Rouse Ball Professor of Mathematics at the University of Oxford, Sir Roger Penrose O.M., who wrote in his book *Shadows of the Mind* (OUP 1994 p 50):

"We shall find ourselves driven towards a *Platonic* view of things. According to Plato, mathematical concepts and mathematical truths inhabit an actual world of their own that is timeless and without physical location. Plato's world is an ideal world of perfect forms, distinct from the real world, but in terms of which the physical world must be understood. It also lies beyond our mental constructions; yet, our minds do have some access to this Platonic realm through an 'awareness' of mathematical forms and our ability to reason about them."

Note that a fairly recent survey of British mathematicians indicated that about 65% of them are Platonists.

With regard to applications, it is true that the Greeks recognized practical mathematics (the mathematics of the architect), but they sharply distinguished it from the mathematics of the philosopher (logically developed geometry) and left us in no doubt that the mathematics of the philosopher was higher, and the mathematics of the architect was lower in value. The division of the sciences into pure sciences and those that are of less worth goes back to Plato, notably his Socratic dialogues *Philebus* and the *Republic*. "Pure Mathematics" is in fact a term that comes directly from *Philebus*. That which was not pure is called "mixed" in these dialogues (even the word "impure is used in the Jowett translation of the *Philebus*). The relevance of this is that the division of mathematics into a higher purer form and a mixed lower form dominated European mathematics for more than two millennia. To check this you need go no further than to look back through the matriculation requirements to be found in old university calendars of the University of Sydney and the University of Melbourne, where you will find this terminology used actively up to the 1950's, at least at the University of Melbourne! There were many more ramifications of the Platonic viewpoint of pure and mixed sciences. Let me be a little provocative and assert that the old NSW split into selective high schools and junior technical high schools (or domestic science schools for girls) is an obvious example of this split.

Above all what is significant for our topic today is that the Greek tradition of despising applications was carried right through to the 1950's, because it was the pure mathematicians who dominated the teaching of mathematics, and they characteristically eschewed applications, as can be seen very easily by looking at the textbooks that were produced. It is not the case that our teaching of mathematics is fully based on what we experience in the creation, to judge by what appears in most contemporary textbooks. Instead a great deal of what we teach is already abstract to the extent that it is not related to the experiential world of the student. In my view, this situation accounts for a great deal of the "Maths is boring" thesis, a provocative example of which can be found in Dr Diana Nisbet's "Ockham's Razor" talk of Sunday 15 April 2001. This message was recently reinforced by the Australian results of the Third International Mathematics and Science Study and reported by Linda Doherty in the Sydney Morning Herald under the headline "Maths lessons are boring and repetitive? Go figure".

There is one major vertical point that I would like to make concerning the Platonist viewpoint, namely, its encouragement of the idea that mathematics gives us an example of certain knowledge is nothing less than idolatrous. To seek for certainty in an aspect of the creation instead of seeking it in God and His faithfulness to His covenant word is to set up an idol, and God will judge all idols and show them to be utterly worthless.

<u>Formalism.</u>

The same remark applies to the second dominant school of mathematical education, namely, that of formalism, and it is worth noting that the unmasking of the religious viewpoint of Platonism and formalism is done very powerfully in Reuben Hersh's very readable book "What is Mathematics Really?" (The only problem with Hersh's book is that he is utterly blind to the religious nature of his own view of mathematics, namely, social constructivism). Recall that formalism holds that there are no mathematical objects in the sense of Platonism. For the formalist mathematics is axioms, definitions and theorems, i.e., formulas. Or if you like, mathematics is a meaningless game. Hersh has a very significant quotation from Bertrand Russell, whom many of you will know as a philosopher, but who made major contributions to mathematical logic. Hersh writes (p 151):

"Russell is frank about his motives, so far as he understands them. In philosophy of science, his leading motive is to establish certainty. In this, he confesses, he's seeking to replace the Christian faith he has rejected. He is also continuing an old tradition: Plato, Descartes, Leibniz, Kant."

While I am unmasking idols connected with mathematics and mathematical education, I should mention the one that according to the outstanding missiologist Lesslie Newbigin is the most powerful one in the secular west. He points out in his very significant book "Foolishness to the Greeks: the gospel and western culture"

“The most obvious fact that distinguishes our culture from all that have preceded it is that it is – in its public philosophy – atheist. The famous reply of Laplace to the complaint that he had omitted God from his system – “I had no need of that hypothesis”—might stand as a motto for our culture as a whole.
“The vision of reality that comes to expression in Laplace’s system still dominates, if I am not mistaken, popular thinking today in spite of all the changes in science itself that have taken place since his time. It assumes that the real world is that which can be “scientifically” explained by laws of cause and effect that can be expressed in mathematical terms.”

I can testify from personal experience that this idol was one that gripped me powerfully when I was a high school student and that only with great difficulty have I been delivered from its clutches.

The switch in mathematical education from Platonism to formalism occurred as a response to the deep crisis in American mathematical education that followed the launching of Sputnik 1 by the Russians. However, ironically, formalism continued the thrust of the Greeks to downplay applications. But if God’s original creation was “very good”, and if we are called to explore all of this good creation, the mathematics with which we engage in the classroom should in principle at least be concerned with the whole of the creation. Let me remind you also of the fact that set theory was introduced into kindergarten classes by formalism through the New Math program. The reason why formalists did that was, to them, set theory was right at the heart of what constitutes mathematics. Every different answer to the question “What is mathematics?” will give rise to a different approach to mathematical education. Hersh gives a further example of how the New Math demonstrated its different answer to this question by introducing (at least into some American grade schools) the game WFF and Proof, for Well Formed Formula and Proof!

To round off the brief definitions of the three major views concerning mathematics, social constructivism is the view that all of knowledge, including mathematics, is based on social norms or linguistic convention (ie having a common language about a subject).

The time has now come to try and give at least a provisional answer from a Christian standpoint to the question “What is mathematics”, and to look at what this could mean for mathematics education in a Christian school.

Over against Platonism, I would say that numbers and geometrical objects are not eternal, but are part of the creation, and therefore had a beginning. Over against formalism we assert that mathematical objects are not arbitrary but are aspects of how concrete things function in the creation. Over against social constructivism we assert that the functioning of mathematical objects is not arbitrary, but is subject to the laws which God has set for them. The investigation of these laws is one of the tasks that God has given to humans, and is what we call “mathematics”. A key part of mathematical activity is the abstracting of numerical and geometric aspects from some concrete thing or event in the creation. (This is the aspect of truth involved in Hersh’s statement that mathematics is a human activity). Because there is no part of the creation that does not function in a mathematical way, mathematics is in principle concerned with every part of the creation. However, created things function in many other ways than mathematically (They can function in physical ways, some can function in biological way, etc). Consequently we do not accept that everything in the creation can be reduced to mathematical functioning, or that the method of mathematical physics is all-powerful and explains everything in the universe. It is very important that we make clear to our students the false nature of that viewpoint. What then might we look for in a mathematics curriculum that takes this view of mathematics seriously? First of all, we would expect that it would be strongly related at all points to the experiential world of the student. With this idea in mind, I looked a couple of years ago in mathematical education libraries and on the internet for whatever possibilities there were of mathematics courses with such an approach. I was happy to find the existence of a very substantial mathematical education institute at the University of Utrecht, the Freudenthal Institute. Professor Hans Freudenthal was a very remarkable mathematician, occupying a front-rank research position in the theory of Lie algebras (a very important field of modern pure mathematics, with enormous ramifications for theoretical physics, in particular quantum mechanics). However, unlike many pure mathematicians, Freudenthal did not relegate applications to an inferior position. Applications were an integral part of his mathematical philosophy, and consequently were an integral part of his approach to mathematical education.

In line with his vision, the staff of the Freudenthal Institute have developed an approach to mathematical education called Realistic Mathematics Education, or RME for short, as their response to the world-wide felt need to reform the teaching and learning of mathematics, partly in response to the American ”New Math” movement but also as a response to the then prevailing Dutch attitude to mathematics education, which was often described as “mechanistic mathematics education”, meaning that methodologically it concentrated on a set of fixed responses to carefully identified situations. In a lecture given in 1998 at Kristiansand, Norway, Marja van den Heuvel-Panhuizen of the Freudenthal Institute stated that:
“The present form of RME is mostly determined by Freudenthal’s (1977) view about mathematics. According to him, mathematics must be connected to reality, stay close to children and be relevant to society, in order to be of human value. Instead of seeing mathematics as subject matter that has to be transmitted, Freudenthal stressed the idea of mathematics as a human activity. Education should give students the “guided” opportunity to “re-invent” mathematics by doing it. This means that in mathematics education, the focal point should not be on mathematics as a closed system but on the activity, on the process of mathematization (Freudenthal, 1968).”

There are several points of Mrs van den Heuvel-Panhuizen’s views to which we will have to come back, from our Christian worldview. At the same time, however, the emphasis on mathematics being close to reality, staying close to children, and relevance to human society should be of importance to us.

The next thing that I found was that RME had not remained imprisoned within its Dutch background, but had become part of a major American effort to renew mathematics education. The National Council of Teachers of Mathematics (NCTM), had recommended that changes be made in mathematics education in the areas of mathematical content, teaching methods, and assessment methods. The National Science Foundation of the United States of America invited Dr Thomas Romberg, the chairperson of the commission for the NCTM Curriculum and Evaluation Standards for School Mathematics to submit a plan to develop a middle-school mathematics program that reflected the vision and the philosophy of the NCTM standards. These standards emphasize the active nature of mathematics and how it enables students to make sense of their world. The way that Dr Romberg began his task was to examine the mathematics curricula of various countries around the world. This search led him to the Freudenthal Institute (FI) in the Netherlands, where the FI staff showed him examples of their real-life mathematics curriculum. Dr Romberg was so impressed that he sought the collaboration of the FI, a collaboration which has resulted in the production of a 40 unit Grade 5 to Grade 8 mathematics program called *Mathematics in Context*, initially published by Encyclopaedia Britannica and now published by Holt, Rinehart and Winston, and which has been extensively field-tested in the United States.
According to a publisher's blurb, "students using *Mathematics in Context* are expected to:
Explore mathematical relationships
Develop and explain their own reasoning and strategies for problem-solving
Use problem-solving tools, such as calculators, appropriately
And
Listen to, understand, and value each other's strategies"

"Mathematics in Context is a core curriculum in which students learn significant mathematics embedded in real-life contexts. Throughout the program, students solve realistic problems using informal strategies that make sense to them. They begin to recognize, understand, and extract the mathematical relationships embedded in a broad range of situations. Students learn how to use the language of mathematics as they investigate topics from the Number, Algebra, Geometry and Statistics strands. They also learn how to use technology, such as graphing calculators and computer software programs."

"Mathematics in Context presents mathematics as a dynamic subject that can best be learned through solving problems embedded in real-life contexts, rather than learning a set of fixed rules and procedures in isolated pieces. The Mathematics in Context units use a variety of realistic contexts as a springboard for motivating students to rediscover rich mathematical concepts. Mathematics in Context recognizes that students come to each unit with prior knowledge and encourages students to solve problems their own way using concrete manipulatives and informal strategies. As students progress, they begin to use more formal strategies. However, students can always fall back on the more concrete, less abstract strategies as needed."

Further on in the same publisher's blurb, we read the following about Assessment and Follow-up:
"Assessment in Mathematics in Context emphasizes the same skills as the activities: reasoning, understanding and communicating. In keeping with the NCTM's Assessment Standards for School Mathematics, the Mathematics in Context program encourages teachers to use a variety of assessment methods to provide them with a more accurate picture of what students know and can do. Such assessment methods include:

informal observations as students work individually or in groups,
students' verbal responses during class discussions,
solutions to problems completed either in class or at home,
group and individual projects,
responses for formal assessments,
and
writing entries in their mathematics journals."

I have seen four of the forty units along with their Teacher guide book, namely, "Take a Chance", from Year 5 Statistics, "Made to Measure" from Year 6 Geometry, "Cereal Numbers" from Year 7 Number, and "Building Formulas" from Year 7 Algebra. I've had a good look at these units and I can confidently say that in every case the mathematics in question is embedded in a real-life context. Obviously many of these contexts relate to North America and would be needed to be adapted for Australian usage. Ideally what I would like to see is an Australian version developed within the Christian School movement. One of the important questions that should then be raised is the extent to which RME is dependent upon a social-constructivism approach to mathematical education. Mrs van den Heuvel-Panhuizen states that "RME has a lot in common with socio-constructivist based mathematics education." (in respect to the view that the students, "instead of being the receivers of ready-made mathematics, are considered as active participants in the teaching-learning process"). You will have noticed that Mrs van den Heuvel-Panhuizen used the words "re-invent" mathematics. On the other hand, you will have noticed that the publisher's blurb about Mathematics in Context says "The Mathematics in Context units use a variety of realistic contexts as a springboard for motivating students to rediscover rich mathematical concepts."

However, the point which I would like to make is that we have here a significantly different model for a mathematics curriculum which gets away from both the Platonist tradition and the formalist tradition, is embedded in the structure of the creation and pays serious attention to the way in which the student begins to make mathematical abstractions and becomes competent in mathematical problem solving. At this stage all I can do is to commend this work to your critical attention and serious evaluation. In closing I would like to mention one very important point that the Freudenthal Institute staff emphasize and which was independently pointed out to me recently by Professor John Mack of the University of Sydney: the reform of mathematics education is difficult and takes time. Though the Freudenthal Institute has been involved in this work of reform for over 30 years, they still regard it as "work under construction". May the Christian

School movement get down to serious work in this field, seeking the blessing of Almighty God.