

# CAN HIGH RESERVES OFFSET WEAK FUNDAMENTALS? A SIMPLE MODEL OF PRECAUTIONARY DEMAND FOR RESERVES

## 1. INTRODUCTION

An important element of the ongoing global macroeconomic imbalances is the large and growing stock-pile of international reserves by Asian economies. To be sure, between end 1990 and 2004 international reserves (excluding gold) in Asia rose from US\$ 400 billion to US\$ 2600 billion (Figure 1)<sup>1</sup>. Part of the motivation for the reserve accumulation may derive from a deep-rooted mercantilist desire by Asian governments to maintain undervalued exchange rates and bolster domestic employment, as well as a general reluctance to forsake hard or soft US dollar pegged regimes (Cavoli and Rajan, 2005)<sup>2</sup>. Apart from these exchange rate objectives which have resulted in rapid reserve accumulation as a side effect, Asian countries have chosen explicitly to build up reserves for precautionary or insurance motives (Bird and Rajan, 2003). For instance, Aizenman and Marion (2003) have noted that the “behavior has changed since the Asian financial crisis”, and go on to suggest that the “recent build-up of large international reserve holdings in a number of Asian emerging markets may represent precautionary holdings” (p. 11).

Precautionary motives for accumulating reserves encompass both crisis management and crisis prevention. The former refers to the role of reserves in reducing the extent of exchange rate (and output) adjustment if a crisis does happen. This in turn could refer either to (a) the ability to finance underlying payments imbalances, or (b) provide liquidity in the face of runs on the currency. Crisis prevention refers

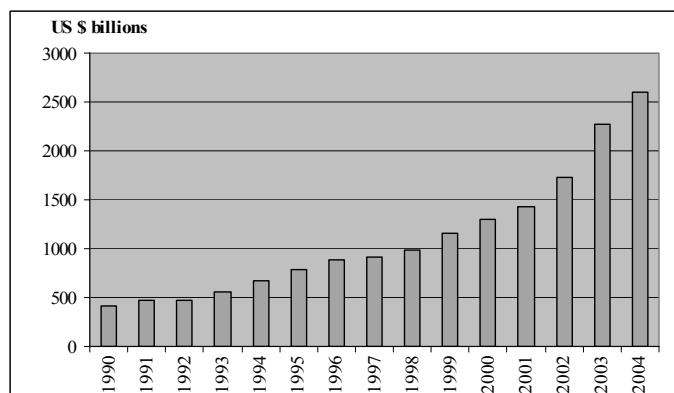
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<sup>1</sup> Asia’s share of global reserves correspondingly rose from about 40 percent in 1990 to 65 percent by 2004.

<sup>2</sup> This apart, it is also possible that countries that have loosened their pegged regimes still choose to hold high reserve levels as they are viewed as a sign of creditworthiness, hence reducing the degree of exchange rate volatility. Some evidence of this thesis is offered by Hviding *et al.* (2004).

broadly to a reduction in the incidence of a crisis. The argument here is simply that, other things equal, high reserves may be viewed as a sign of strength of an economy, thus reducing the chances of a run against the currency. Indeed, many studies have confirmed that high reserves to short term debt or money supply ratios have consistently stood out as being robust predictors of a crisis (Bird and Rajan, 2003, De Beaufort Wijnholds and Kapetyn, 2001, and Willett *et al* 2004).

FIGURE 1 – *International Reserves in Asia, 1990-2004*



Source: International Financial Statistics

Some have even suggested that sufficiently high levels of reserves can fully offset weak fundamentals (Sachs *et al.*, 1996). Counterbalancing these precautionary motives for holding reserves are their high opportunity costs which arise from substituting high yielding domestic assets for lower yielding foreign ones. These costs can be proxied as the difference between the domestic marginal product of capital and the returns obtained on the reserve assets (usually US Treasuries)<sup>3</sup>.

This paper has a rather modest objective. It attempts to develop a simple optimizing model to determine the optimal reserve holdings

<sup>3</sup> Two caveats are in order. One, it is sometimes noted that reserves could be used to pay down external debt. The difference of the interest rate paid on the external debt and from that earned on reserve assets could be a proxy for opportunity cost of holding reserves. Two, another set of costs of persistent reserve accumulation arises due to the inflationary consequences of excess liquidity and/or the costs of mopping up the liquidity, i.e. sterilization (for instance, see Kletzer and Spiegel, 2004).

by a country looking to minimize the net costs of holding reserves. In so doing the paper also attempts to determine the validity of the Sachs *et al.* (1996) assertion that sufficiently high levels of reserves can compensate for weak fundamentals.

The remainder of this paper is organized as follows. The next section outlines the basic structure of the model and solution. Section 3 discusses the nexus between weak fundamentals and optimal reserve size. Section 4 offers some concluding observations.

## 2. THE MODEL

### 2.1 Basic Structure and Assumptions

The basic model structure is fairly simple and intuitive. We assume a risk averse central bank's aim is to minimize the expected total costs to the economy from holding international reserves ( $R$ ) which is its choice variable<sup>4</sup>.

As noted, the major precautionary benefits from holding reserves are twofold. One, a stockpile of reserves may reduce the probability of a crisis occurring in the first instance, i.e. crisis prevention role. Two, reserves help reduce the adjustment costs if a crisis does occur, i.e. crisis management role.

In other words:

$$TC = PC_c + RC_R \quad (1)$$

$TC$ : expected total costs.

$R$ : level of international reserves.

$C_c$ : unit cost associated with the crisis, measured as the output loss, viz. the difference of the output levels between normal times and crises.

$C_R$ : unit opportunity cost of holding reserves. We assume this to be constant.

$P$ : probability of crises which is a function of  $R$  as well as a vector of weak fundamentals ( $X$ ). In addition,  $P_X > 0$  and  $P_R < 0$ .

The output loss ( $C_c$ ) is assumed to be the difference of the output levels between normal times and crisis.

$$C_c = Y_N - Y_C \quad (2)$$

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<sup>4</sup> For an early cost-benefit analysis on the issue of optimal reserves, see Bassat and Gottlieb (1992).

$Y_N$ : the output level in normal times; and  $Y_C$ : the output level in crises times.

We assume, for simplicity, that the only input of production is capital ( $K$ ):

$$Y=F(K) \quad (3)$$

where:  $\frac{\partial F}{\partial K} > 0$ ,  $\frac{\partial^2 F}{\partial K^2} < 0$ .

We need to make explicit the costs of a crisis. Assume that during normal times  $K = \bar{K}$ . To maintain a degree of generality, we assume that a crisis -- bad state of nature -- acts as a negative supply shock in the sense that either the extent of capital stock deteriorates, or the average productivity of capital declines ( $A$ )<sup>5</sup>. However, for a given crisis, the bad state of nature is inversely related to the amount of reserves. In other words, the extent of impact of the bad state of nature is lower the higher is the stock of reserves. So:

$$K = \begin{cases} \bar{K}, & \text{in normal times} \\ A(R)\bar{K}, & \text{in crises} \end{cases} \quad (4)$$

where,  $0 < A(R) < 1$  and  $A_R > 0$ <sup>6</sup>.

Plugging eqs. (3) and (4) into (2), we can express the output loss as a function of reserves:

$$C_C = Y_N - Y_C = F(\bar{K}) - F[A(R)\bar{K}] \quad (5)$$

From eq. (5) we have:

$$\frac{\partial C_C}{\partial R} = -F'[A(R)\bar{K}] * \bar{K} * \frac{\partial A}{\partial R} < 0. \quad (6)$$

Eq. (6) reveals a negative relationship between the reserve holding and output loss during a crisis.

## 2.2 Model Solution

The central bank minimizes the loss function (eq. 1) so as to choose the optimal reserve. The first order condition of this

<sup>5</sup> Alternatively, one could define a crisis as something that reduces the extent of capital reversal (Kim *et al.*, 2005).

<sup>6</sup> If we interpret the shock in terms of capital reversals ( $CR$ ), viz. the difference of capital flows in crisis and previous inflows, then  $CR = \bar{K}_C - \bar{K} = [A(R) - 1]\bar{K}$ .

minimization problem is:

$$\frac{\partial P}{\partial R} * C_C + \frac{\partial C_C}{\partial R} * P + C_R = 0. \quad (7)$$

For concreteness, we make use of some specific functional forms. Let the probability function of crisis be:

$$\begin{aligned} P &= P(X; R) \\ &= \exp[-R / X]. \end{aligned} \quad (8)$$

Following Sachs *et al.* (1996),  $X$  usually consists of at least four variables, viz. current account deficit ( $CAD$ ), lending boom ( $LB$ ), real exchange rate appreciation ( $RER$ ), and the size of external debt ( $STD$ )<sup>7</sup>. The probability function reveals that with the accumulation of higher levels of reserves ( $R$ ), the probability of crisis will converge to 0. If the level of reserves is close to 0, the probability of crisis will increase, peaking at 1. Meanwhile, if the weak fundamentals ( $X$ ) are close to 0, the probability of crisis will decrease to 1; and if the weak fundamentals ( $X$ ) are significantly high, the probability of a crisis will increase to 1.

From eq. 8 we have:

$$\frac{\partial P}{\partial R} = (-1 / X)[\exp(-R / X)] = -P / X \quad (9)$$

Plugging eq. 9 into eq. 7 and solving for P derives:

$$P = \frac{C_R}{\frac{C_C}{X} - \lambda}, \quad \text{while } \lambda = \frac{\partial C_C}{\partial R} \quad (10)$$

From eq. 7 and 10 we have:

$$\exp(-R / X) = \frac{C_R}{\frac{C_C}{X} - \lambda} \quad (10^1)$$

In order to solve for  $R^*$ , assume

$$Y = K^\alpha \text{ and } A(R) = 1 - \exp(-R)$$

Note that when  $R \rightarrow 0, A(R) = [1 - \exp(-R)] \rightarrow 0$ ; when  $R \rightarrow \infty, A(R) \rightarrow 1$

$$C_C = Y_N - Y_C = F(\bar{K}) - F[A(R)\bar{K}] = \bar{K}^\alpha [1 - (1 - e^{-R})^\alpha] \quad (11)$$

<sup>7</sup> For instance,  $X = \alpha CAD + \beta LB + \gamma RER + \eta STD$ . However, in view of the possible tradeoffs between the various variables, there is not yet a clear indication of the best way of interacting them to come up with a suitable vector of weak fundamentals (Willett *et al.*, 2004).

$$\lambda = \frac{\partial C_C}{\partial R} = -a\bar{K}^a e^{-R} (1 - e^{-R})^{a-1} \quad (12)$$

Plug eqs. 11 and 12 into eq. 10 and rearranging derives:

$$\frac{e^{-R/X} \bar{K}^a [1 - (1 - e^{-R})^a]}{X} + a\bar{K}^a e^{-R-(R/X)} (1 - e^{-R})^{a-1} = C_R \quad (13)$$

The left hand side of eq. 13 can be interpreted as the marginal benefit of holding reserves, while the right hand side is the marginal cost of reserves. In other words, an optimizing central bank will continue to build up reserves as long as the marginal benefits of doing exceed the marginal costs (opportunity costs). While this result is intuitive, the contribution of the simple model is to flesh out the factors that impact the marginal benefits which in turn allow us to analyze the nexus between weak fundamentals and reserve holdings. We elaborate on this issue in the next section.

### 3. FINDINGS AND IMPLICATIONS

While the right hand side of eq. 13 is assumed constant, the left hand side. is decreasing in  $R^8$ . Given this, we have the following proposition -- *the sufficient condition for the existence of  $R^*$  is  $X \leq \frac{\bar{K}^a}{C_R}$ . Why?*

We can set the domain of the left hand side as  $[0, \infty)$ . If we set the initial  $R$  as 0, then, the output loss  $C_C = \bar{K}^a$ , and the marginal output loss,  $\lambda=0$ . Thus, the left hand side of eq. 13 is reduced to  $\frac{\bar{K}^a}{X}$ . Therefore, the condition that  $X \leq \frac{\bar{K}^a}{C_R}$  can ensure there is at

least one level of  $R$  such that the left hand side is greater than or equal to right hand side of eq. 13.

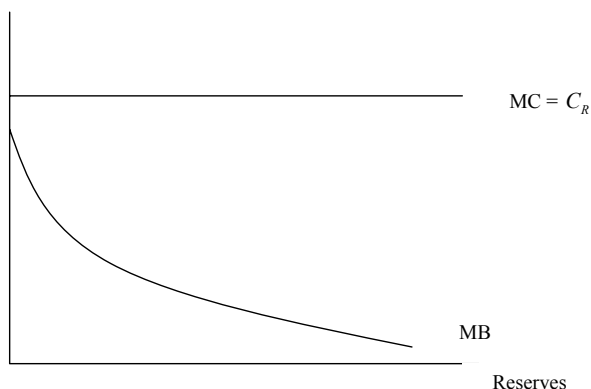
The proposition can be reinterpreted as follows. If the fundamentals (X) are sufficiently weak or the opportunity costs of holding reserves are sufficiently high, such that  $X \leq \frac{\bar{K}^a}{C_R}$ , there may

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<sup>8</sup> Taking the partial derivative w.r.t.  $R$  we see that as  $R$  rises, the lower is  $\frac{e^{-R/X} \bar{K}^a}{X}$ ,  $[1 - (1 - e^{-R})^a]$ ,  $a\bar{K}^a e^{-R-(R/X)}$ , and  $(1 - e^{-R})^{a-1}$ . All these terms are positive. Therefore, with the increase of  $R$ , all the terms on the left hand side of eq. 13 decrease. In other words, the left hand side is a decreasing function of  $R$ .

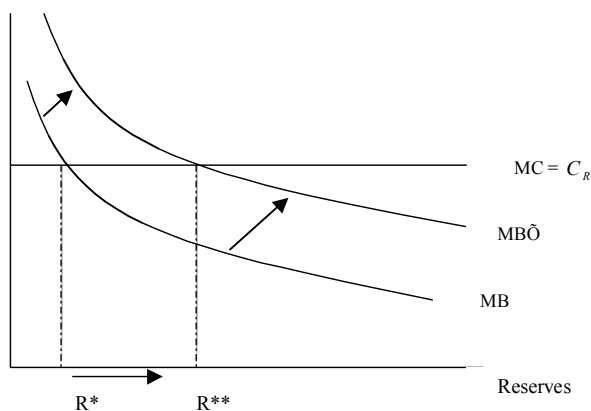
not be any interior solution to  $R^*$ . In other words, for extremely weak fundamentals no amount of reserves can help prevent a crisis from occurring (Figure 2).

FIGURE 2 – Reserves Insufficient to Offset Weak Fundamentals



For the case where  $X \leq \frac{\bar{K}^a}{C_R}$ , there is an interior solution for  $R^*$ . Worsening fundamentals (i.e. rising  $X$ ) will lead to higher probability of crisis. This in turn, increases the marginal benefit of reserve holdings at any given reserve level. Therefore, the MB schedule will shift up from  $R^*$  to  $R^{**}$  (see Figure 3)<sup>9</sup>. So generally,

FIGURE 3 – Worsening Fundamentals Compensated for by Higher Reserves



<sup>9</sup> Annex 1 derives the specific conditions under which the  $MB$  curve rises with  $X$ .

as fundamentals get weaker, countries need to hold correspondingly more reserves, and high reserves can offset weak fundamentals only if the fundamentals are not “too weak”.

#### 4. CONCLUDING REMARKS

This paper has explored the issue of optimal reserve holdings by a central bank within a context of a simple analytical model. An important limitation of the model arises from the assumption of a constant opportunity cost of reserves. More realistically, insofar as these costs can be proxied as the opportunity cost, it is important to consider the impact of changes in the capital stock and production on the marginal costs of reserve holdings. This notwithstanding, the model suggests that in general, high reserves can help offset weak fundamentals. However, if fundamentals are sufficiently weak no level of reserves will be able to offset the weak fundamentals. In other words, for “hopelessly weak” fundamentals, a crisis is inevitable and reserves cannot act as a substitute for domestic policy reforms and adjustments<sup>10</sup>. Conversely, if fundamentals are “sufficiently strong”, a crisis will never occur. However, if fundamentals are within a certain range – zone of vulnerability – other things equal, higher levels of reserves may help offset the negative impact of weak fundamentals. With fundamentals in the vulnerable zone, high reserves could have a powerful effect in protecting against crises. This also suggests that reserve needs should be related to the state of fundamentals in a non-linear manner. While this may contradict the conclusion of Sachs *et al.* (1996), it is broadly in line with the critique by Willett *et al.* (2004), and is also consistent with the escape clause based second-generation models of currency crisis (Obstfeld, 1994, 1996 and Rajan, 2001).

Returning to the issue of reserve stockpiling in Asia. The fact that a number of Asian countries are consciously looking to use part of their accumulated reserves to finance physical infrastructure (e.g. India and Thailand) or strengthen their financial institutions (e.g. Korea and China), seems to indicate that they have reached a level at which their perceived marginal benefits have been outweighed by

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<sup>10</sup> Indeed, first generation crisis models imply that if fundamentals are sufficiently weak such that a crisis is inevitable, reserve levels should only influence the timing of crises not whether they occur.

their marginal costs. This in turn suggests that the recent build up of reserves in Asia has been more due to exchange rate motivations (i.e. mercantilism or general commitment to pegged regimes which are undervalued) rather than a conscious attempt to buy “insurance cover”.

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## ANNEX 1

This Annex derives the conditions under which the Marginal benefit ( $MB$ ) curve rises with  $X$ <sup>11</sup>. Taking the first derivative of the left hand side of eq. 13 w.r.t.  $X$  derives:

$$MB = e^{-R/X} \left\{ \frac{\bar{K}^a [1 - (1 - e^{-R})^a]}{X} + a\bar{K}^a e^{-R} (1 - e^{-R})^{a-1} \right\}$$

To simplify the notations, let  $y = \bar{K}^a [1 - (1 - e^{-R})^a]$  and  $z = a\bar{K}^a e^{-R} (1 - e^{-R})^{a-1}$

$$\begin{aligned} \frac{\partial MB}{\partial X} &= \frac{R e^{-R/X}}{X^2} \left( \frac{y}{X} + z \right) - e^{-R/X} \left( \frac{y}{X^2} \right) \\ &= e^{-R/X} \left( \frac{Ry}{X^3} + \frac{Rz}{X^2} \right) - e^{-R/X} \left( \frac{y}{X^2} \right) \\ &= e^{-R/X} \left( \frac{yR + XRz - yX}{X^3} \right) = e^{-R/X} \left[ \frac{y(R - X)}{X^3} + \frac{Rz}{X^2} \right] \end{aligned}$$

If  $R > \frac{yX}{y + Xz}$ , then,  $y(R - X) + XRz > 0$ , and  $\frac{\partial MB}{\partial X} > 0$ .

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<sup>11</sup> While not shown here, the impact of a change of  $X$  on the slope of the  $MB$  curve (i.e.  $\partial(\partial MB/\partial R)/\partial X$ ) is ambiguous.

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