The RC Time Constant

Objectives

When a direct-current source of emf is suddenly placed in series with a capacitor and a resistor, there is current in the circuit for whatever time it takes to fully charge the capacitor. In a similar manner, there is a definite time needed to discharge a capacitor that has previously been charged. There is a characteristic time associated with either of these processes, called the "RC time constant," whose value depends on the value of the resistance $R$ and the capacitance $C$. In this laboratory, series combinations of a power supply, a capacitor, and resistors will be used to accomplish the following objectives:

1. Demonstration of the finite time needed to discharge a capacitor
2. Measurement of the voltage across a resistor as a function of time
3. Determination of the RC time constant of two series RC circuits
4. Determination of the value of an unknown resistor from measurements made on a series RC circuit with an unknown resistance in parallel with the voltmeter

Equipment List

1. Voltmeter (at least 10-MΩ input impedance, preferably digital readout)
2. Direct-current power supply (20 V)
3. Laboratory timer
4. High-quality capacitor (5 µF)
5. Resistor (10 MΩ to serve as an unknown)
6. Single-pole, double-throw switch
7. Assorted connecting leads

Theory

Consider the circuit shown in Figure 1 consisting of a capacitor $C$, a resistor $R$, a source of emf $\varepsilon$, and a switch $S$. If the switch $S$ is thrown to point $A$ at time $t = 0$ when the capacitor is initially uncharged, charge begins to flow in the series circuit consisting of $\varepsilon$, $R$, and $C$ and flows until the capacitor is fully charged. It can be shown that the current $I$ starts at an initial value of $\varepsilon/R$ and decreases exponentially with time. The charge $Q$ on the capacitor, on the other hand, begins at zero and increases exponentially with time until it becomes equal to $C\varepsilon$. The equations that describe those events are

$$Q = C\varepsilon (1 - e^{-t/RC}) \quad \text{and} \quad I = \varepsilon/R \ e^{-t/RC} \quad (1)$$

![Figure 1 Simple Series RC Circuit](image-url)
The quantity $RC$ is called the “time constant of the circuit,” and it has units of seconds if $R$ is expressed in ohms and $C$ is expressed in farads. After a period of time that is long compared to the time constant $RC$, the term $e^{-t/RC}$ becomes negligibly small. When this is true the equations above predict that the charge $Q$ is equal to $C\varepsilon$, and the current in the circuit is zero.

If switch $S$ is now thrown to position $B$, which effectively takes $\varepsilon$ out of the circuit, the capacitor discharges through the resistor. Therefore, the charge on the capacitor and the current in the circuit both decay exponentially while the capacitor is discharging. The equations that describe this discharging process are

$$Q = C\varepsilon e^{-t/RC}$$
$$I = \frac{\varepsilon}{R} e^{-t/RC}$$

(2)

The equation for the current could be written with a negative sign because the current in the discharging case will be in the opposite direction from the current in the charging case. The magnitude of the current is the same in both cases. Although the above discussion has included both the case of charging and discharging a capacitor, this laboratory will only investigate the process of discharging a capacitor.

![Figure 2](image)

**Figure 2** An RC circuit using a voltmeter as the resistance.

Consider the circuit shown in Figure 2 consisting of a power supply of emf $\varepsilon$, a capacitor $C$, a switch $S$, and a voltmeter whose input impedance is $R$. If initially the switch $S$ is closed, the capacitor is charged almost immediately to $\varepsilon$, the voltage of the power supply. When the switch is opened the capacitor discharges through the resistance of the meter $R$ with a time constant given by $RC$. With the switch open the only elements in the circuit are the capacitor $C$ and the voltmeter resistance $R$; thus the voltage across the capacitor is equal to the voltage across the voltmeter. The voltage across the capacitor is given by $Q/C$, and the voltage across the voltmeter is given by $\varepsilon/R$. Solving 2 for those quantities leads in both cases to

$$V = \varepsilon e^{-t/RC}$$

(3)

Equation 3 stands either for the voltage across the voltmeter or the voltage across the capacitor as a function of time. Dividing both sides of equation 3 by $\varepsilon$ and taking the reciprocal of both sides of the equation leads to
\[ \frac{\epsilon}{V} = e^{-t/RC} \]  

(4)

Taking the natural logarithm of both sides of equation 4 leads to the following:

\[ \ln \left( \frac{\epsilon}{V} \right) = \left( \frac{1}{RC} \right) t \]  

(5)

Equation 5 states that there is a linear relationship between the quantity \( \ln(\epsilon/V) \) and the time \( t \) with the quantity \( (1/RC) \) as the constant of proportionality. Therefore, if the voltage across the capacitor is determined as a function of time, a graph of \( \ln(\epsilon/V) \) versus \( t \) will give a straight line whose slope is \( (1/RC) \). Thus, \( RC \) can be determined, and if \( C \) is known, the voltmeter resistance \( R \) can be determined.

\[ R_U = \frac{RR_t}{R-R_t} \]  

(6)

Therefore, a measurement of the capacitor voltage as a function of time will produce a dependence like that given by equation 5 except that the slope of the straight line will be \( (1/R_tC) \). Thus, if \( C \) is known and \( R_tC \) is found from the slope, then \( R_t \) can be determined. Using equation 6, \( R_U \) can be found from \( R \) and the value just determined for \( R_t \).

Figure 3  RC circuit using voltmeter and \( R_U \) in parallel as resistance.
EXPERIMENTAL PROCEDURE – RESISTANCE OF THE VOLTMETER

1. Construct a circuit such as the one in Figure 2 using the capacitance decade box (set to 5 µF), the voltmeter (acting also as your resistor), and the power supply. **Have the circuit approved by your instructor before turning on any power.** Record the value of the capacitor (5 µF) in Data Table 1.

2. Close the switch, and while reading its voltage on the voltmeter, adjust the power supply emf \( \varepsilon \) to 20 V (or as close as your power supply will go). Record the value of \( \varepsilon \) in Data Table 1.

3. Open the switch and simultaneously start the timer. The person using the timer should also open the switch.

4. The voltmeter reading will fall as the capacitor discharges. Let the timer run continuously, and for eight predetermined values of the voltage, record the time \( t \) on the timer when the voltmeter reads these voltages. A convenient choice for voltages at which to measure the time would be increments of 10%. Since \( \varepsilon = 20.0 \) V, record the time when the voltage is 18.0 V, 16.0 V, 14.0 V, etc down to 4.0 V. Record the values of the voltage at which the time is to be read in Data Table 1 as \( V \).

5. Record the values of \( t \) at which each value of \( V \) occurs in Data Table 1 under Trial 1.

6. Repeat steps 2 through 4 two more times, recording the values of \( t \) under Trials 2 and 3 in Data Table 1.

EXPERIMENTAL PROCEDURE – UNKNOWN RESISTANCE

1. Construct a circuit such as the one in Figure 3 using the same capacitor used in the last circuit and the unknown resistor supplied (the decade resistance box set to 10 M \( \Omega \)). Close the switch and adjust the power supply voltage to the same value used in the last procedure.

2. Repeat steps 2 through 6 of the procedure above, but record all values in the appropriate places in Data Table 2.

CALCULATIONS – UNKNOWN CAPACITANCE

1. Calculate the values of \( \ln(\varepsilon/V) \) and record them in Calculations Table 1.

2. Calculate the mean \( \bar{t} \) and the standard deviation \( \sigma \) for the three trials of the time \( t \) at each voltage and record them in Calculations Table 1.

3. Graph the quantity \( \ln(\varepsilon/V) \) vs. \( \bar{t} \). Draw the line of best fit. Determine the slope and the y-intercept.

4. The value of the slope is equal to \( 1/RC \). Record the value of the slope in Calculations Table 1. (The units of \( 1/RC \) are s\(^{-1}\).)

5. Calculate \( RC \) as the reciprocal of the slope. Record the value of \( RC \) in Calculations Table 1. (The units of \( RC \) are s.)
6. Using the value of $RC$ and the value of $C$, calculate the value of the resistance of the voltmeter $R$ and record it in Calculations Table 1.

**CALCULATIONS – UNKNOWN RESISTANCE**

1. Calculate the values of $\ln(\varepsilon/V)$ and record the values in Calculations Table 2.

2. Calculate the mean $\bar{t}$ and the standard deviation $\sigma_t$ for the three trials of the time $t$ at each voltage and record them in Calculations Table 2.

3. Graph the quantity $\ln(\varepsilon/V)$ vs. $\bar{t}$. Draw the line of best fit. Determine the slope and the y-intercept.

4. The value of the slope of this fit is equal to $1/R_tC$. Record the value of the slope in Calculations Table 2. (The units of $1/R_tC$ are s$^{-1}$.)

5. Calculate the value of $R_tC$ as the reciprocal of the slope. Record the value of $R_tC$ in Calculations Table 2. (The units of $R_tC$ are s.)

6. Using the value of the capacitance $C$ and the value of $R_tC$, calculate the value of $R_t$ and record it in Calculations Table 2.

7. Using equation 6, calculate the value of the unknown resistance $R_U$ from the values of $R_t$ and $R$ (determined in the first part). Record the value of $R_U$ in Calculations Table 2.

8. Calculate the percent error of $R_U$ using the value of 10 MΩ (set on the decade resistance box) as the accepted value.
LABORATORY REPORT

Data Table 1

<table>
<thead>
<tr>
<th>V (V)</th>
<th>t₁ (s)</th>
<th>t₂ (s)</th>
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\[ \mathcal{E} = \frac{V}{C} \]

Calculations Table 1

<table>
<thead>
<tr>
<th>ln (\mathcal{E}/V)</th>
<th>\bar{\tau} (s)</th>
<th>\sigma_1 (s)</th>
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Intercept =
Slope = \frac{s}{s^1}
RC = \frac{s}{s}
R = \Omega
### Data Table 2

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<th>$V$ (V)</th>
<th>$t_1$ (s)</th>
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<td>$\mathcal{E}$ =</td>
<td>$V$</td>
<td>$R$ =</td>
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### Calculations Table 1

<table>
<thead>
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<th>$\bar{\varepsilon}$ (s)</th>
<th>$\sigma_1$ (s)</th>
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<td>$R_{U}$ =</td>
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% Error ____________________________