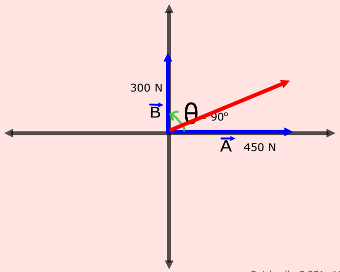
9.3 Addition of Vectors by Components

<Preview>

Given: Two vectors $\vec{A}$ and $\vec{B}$ where θ = 90 °.

Find the resultant vector $\vec{R}$



Use the Pythagorean Theorem to find the R

$a^2 + b^2 = c^2$

$300^2 + 450^2 = c^2$

$540.8 = c$

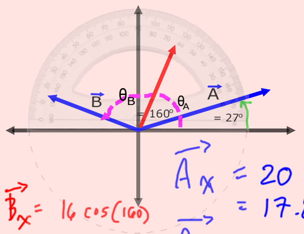
Use tan to find the direction of R

$\tan\theta_R = \frac{300}{450}$

$\theta_R = 33.69°$

Quickpoll: P.271, #6...Magnitude...direction

In this special case, the vectors given are already positioned where θ is right angle....but what if they are given at a different angle?



Given:

$\vec{A} = 20$

$\theta_A = 27°$

$\vec{B} = 16$

$\theta_B = 160°$

$\vec{A}_x = 20\cos(27°)$
$= 17.82$

$\vec{A}_y = 20\sin(27°)$
$= 9.08$

$\vec{B}_x = 16\cos(160)$
$= -15.04$

$\vec{B}_y = 16\sin(160)$
$= 5.47$

$\vec{A}_x + \vec{B}_x = 2.78 = \vec{R}_x$

$\vec{A}_y + \vec{B}_y = 14.55 = \vec{R}_y$

$\tan^{-1}\theta_R = \left|\frac{R_y}{R_x}\right| =$

$\vec{R} = \sqrt{\vec{R}_x^2 + \vec{R}_y^2}$

$\vec{R} = \sqrt{2.78^2 + 14.55^2}$

$\vec{R} = 14.81$

$\theta_R = 79.2°$

Now let's put more details into this process...

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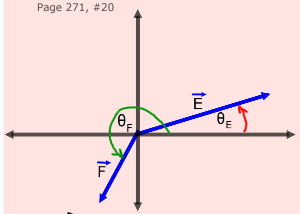

Given:

$\vec{E} = 1.653$

$\theta_E = 36.37°$

$\vec{F} = 0.9807$

$\theta_F = 253.06°$

$\vec{E}_x = 1.653\cos(36.37) = 1.33$

$\vec{E}_y = 1.653\sin(36.37) = 0.98$

9.3 Addition of Vectors by Components

<Preview>

Now let's put more details into this process...

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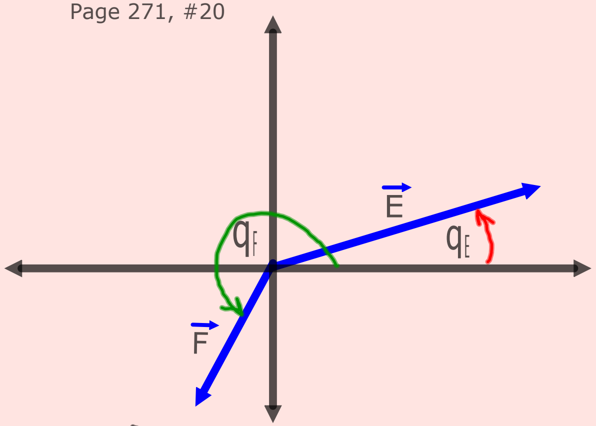


Given:

$\vec{E} = 1.653$

$q_E = 36.37°$

$\vec{F} = 0.9807$

$q_F = 253.06°$

Use Reference Angle instead

253.06 − 180

$\vec{E_x} = 1.653 \cos(36.37) = 1.33$

$\vec{E_y} = 1.653 \sin(36.37) = 0.98$

$\vec{F_x} = 0.9807 \cos(73.06) = 0.29$

$\vec{F_y} = 0.9807 \sin(73.06) = 0.94$

$\vec{R_x} = \vec{\Sigma x} + \vec{F_x} = 1.62$

$\tan\theta = \left| \frac{R_y}{R_x} \right|$

$\vec{R_y} = \vec{E_y} + \vec{F_y} = 1.92$

$\tan\theta = \frac{1.92}{1.62}$

$\vec{R} = \sqrt{1.62^2 + 1.92^2} = 2.51$

$\theta_R = 0.02$

9.3 Addition of Vectors by Components (continued)

Review:

1. Given: a vector A

Find: Components
$A_x = A\cos\theta_A$
$A_y = A\sin\theta_A$

2. Given: A_x, A_y

Find: $\vec{A}$

$A = \sqrt{A_x^2 + A_y^2}$

$\theta_{ref} = \tan^{-1}\left|\frac{A_y}{A_x}\right|$

θ_A be careful! θ_A depends on what quadrant!

3. Given: $\vec{A}, \vec{B}$ Find: $\vec{R}$

Vector	Magnitude	Angle	x-component	y-component
$\vec{A}$	✓	✓	$A_x = A\cos\theta_A$	$A_y = A\sin\theta_A$
$\vec{B}$	✓	✓	$B_x = B\cos\theta_B$	$B_y = B\sin\theta_B$
$\vec{R} = \vec{A} + \vec{B}$	?	?	?	?

$R = \sqrt{R_x^2 + R_y^2}$

$\theta_{ref} = \tan^{-1}\left|\frac{R_y}{R_x}\right|$

$R_x = A_x + B_x$

$R_y = A_y + B_y$

NOTE! All of the examples on the following pages are completed by using the formulas listed above, but all of the works may or may not be listed.

9.3 Ex #18

$\vec{A} = 6.89, \quad q_A = 123°$

$\vec{B} = 29, \quad q_B = 260°$

Vector	Magnitude	Angle	x-component	y-component
$\vec{A}$	6.89	123°	$A_x = 6.89 \cos(123)$ $= -3.75$	$A_Y = 6.89 \sin(123)$ $= 5.78$
$\vec{B}$	29	260°	$B_x = 29 \cos(260)$ $= -5.04$	$B_Y = 29 \sin(260)$ $= -28.56$
$\vec{R} = \vec{A} + \vec{B}$	$R = \sqrt{R_x^2 + R_y^2}$ $R = \sqrt{-8.79^2 + -22.78^2}$ $R \approx 24.42$	$q_{ref} = \tan^{-1}\left\lvert\frac{R_y}{R_x}\right\rvert$ $q_{ref} = \tan^{-1}\left\lvert\frac{-22.78}{-8.79}\right\rvert$ $q_{ref} = 68.9°$	$R_x = A_x + B_x$ $= -3.75 + (-5.04)$ $= -8.79$	$R_Y = A_Y + B_Y$ $= 5.78 + (-28.56)$ $= -22.78$

$q_R = 180 + 69$

(Since R is in Q III)

$q_R = 249°$

Ex #22...3 vectors

$\vec{R} = 6300, \quad q_R = 189.6°$

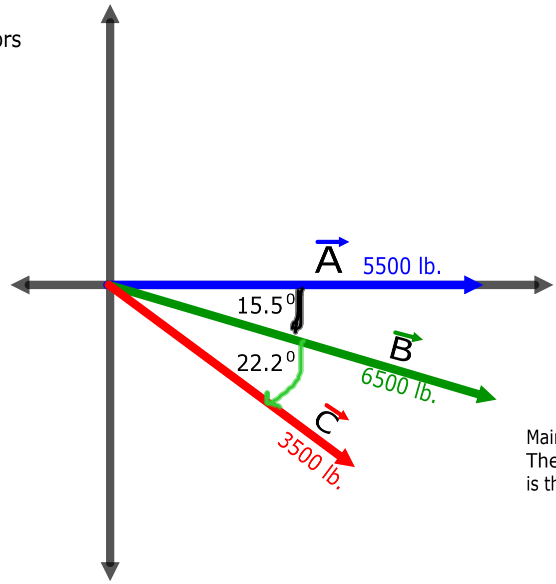
$\vec{F} = 1760, \quad q_F = 320.1°$

$\vec{T} = 3240, \quad q_T = 75.4°$

Vector	Magnitude	Angle	x-component	y-component
$\vec{R}$	6300	189.6°	$R_x = -6211.78$	$R_Y = -1050.64$
$\vec{F}$	1760	320.1°	$F_x = 1350.21$	$F_Y = -1128.95$
$\vec{T}$	3240	75.4°	$T_x = 816.7$	$T_Y = 3135.38$
$\vec{R} + \vec{F} + \vec{T} = \vec{G}$	$G = \sqrt{G_x^2 + G_y^2}$ $G \approx 4156.26$	$q_{ref} = \tan^{-1}\left\lvert\frac{G_y}{G_x}\right\rvert$ $q_{ref} = 13.29°$ $q_G = 180 - 13.29$ (Since R is in Q II)	$G_x = -4044.87$	$G_Y = 955.79$

$q_G = 166.71$

Ex #27...3 vectors



Main Point that people miss...
The SUM of the TWO angles
is the reference angle for Vector C!

Vector	Magnitude	Angle	x-component	y-component
$\vec{A}$	5500	0^{0}	A_x = 5500	A_y =0
$\vec{B}$	6500	344.5^{0}	B_x = 6263.6	B_y =-1737.05
$\vec{C}$	3500	322.3^{0}	C_x = 2769.28	C_y =-2140.34
$\vec{R} = \vec{A} + \vec{B} + \vec{C}$	15041.24	$\theta_{ref} = 14.94^{0}$ $\theta_R = 345.06^{0}$	R_x = 14532.88	R_y =-3877.39

$$\theta_{ref} = \tan^{-1}\left(\frac{3877.39}{14532.88}\right)$$

14.94^{0}

NOTE! Since all of these vectors reside in Q IV, the reference angles are subtracted from 360 to get the angles for the vectors in *standard position* .

$\theta_R =$