COMPLEXITY OF EARTHQUAKES: LEARNING FROM SIMPLE MECHANICAL MODELS

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Abstract:

It has long been recognized that earthquakes exhibit similarity at many length scales which suggests fractal characteristics of earthquake ruptures. One of the well known manifestations of earthquake complexity is the fractal nature of the Gutenberg-Richter law. Moreover, there is growing evidence that earthquakes ruptures are very complex processes; as the resolution of source inversions has increased, so has the temporal and spatial complexity of the rupture. What are the key physical parameters that control this complexity? Many methods have been tailored in an attempt to uncover the details of the complex rupture processes but due to different numerical and computational limitations scientists could not go beyond a few orders of magnitude on both of the spatial and temporal scales. Statistical methods provide an alternative way to overcome the difficulties associated with numerical models of spontaneous dynamic ruptures. Here, we present an outline for an innovative approach that aims at reproducing the different earthquake complex scaling laws using methods of statistical mechanics with the ultimate goal of finding the scaling behavior of the strength of the Earth's crust. The emphasis of this presentation is on exploring the conditions that might lead to complexity in earthquakes using simple mechanical models similar to the Burridge-Knopoff spring-block-slider models. Although there are limitations in the usage of spring-block-sliders to interpret real earthquakes, these models have the advantage that they are computationally efficient. This allows us to explore the nature of complexity that is produced by different types of models of dynamic friction. We show that chaotic behavior occurs for friction models with hyperbolic velocity or slip weakening. We would briefly explain why this might be suggestive for assessing complexity using fractals and introduce an outline for an alternative method that could be a surrogate for solving the full equilibrium equations and reproduce the rough earthquake statistics

Motivation:

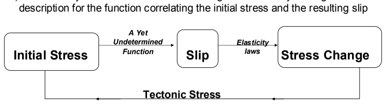
1. Earthquakes exhibit complexity in both spatial and temporal domains.
2. Earth might be failing at all length scales as suggested by the power law nature of the "Gutenberg-Richter Law".
3. The internal stress in the lithosphere might then be described by a law which has no characteristic length scale (i.e. a fractal law)
4. Could we verify this through modeling cycles of earthquakes and investigate the resulting scaling laws?

a) *We* could go to the 3D continuum models and try to solve the equilibrium equations numerically as it is usually done, *BUT:*

Δx = 100 m ⟶ 1 day run time for a sequence of events
Δx = 1 m ⟶ ~10^{12} days

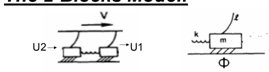
Mission Impossible !!!

b) Alternatively we could search for a surrogate method by finding a statistical description for the function correlating the initial stress and the resulting slip



To find a clue for this surrogate method we are considering some simple mechanical models like Spring Block Slider models which can approximate some aspects of earthquake rupture. These models lack the long range interaction characterizing 3D elasticity models as any block can only interact with its adjacent one through the connecting spring but on the other hand are computationally efficient and can produce rough earthquake statistics

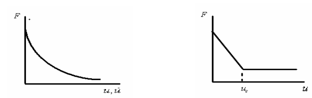
The 2-Blocks Model:



The purpose here was to explore what class of friction laws might lead to complexity in such low dimensional dynamical systems.

The choice of the two-blocks system was because its phase space has the minimum dimension that allows chaos to exist.
The friction law is a function of slip or slip velocity or both.

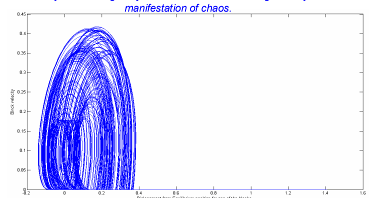
Two major types of friction law are considered here: The hyperbolic laws with no length scale and the traditional model which has a characteristic slip weakening distance and weakening slope



(Hyperbolic Weakening Friction) (Traditional Slip Weakening Friction)

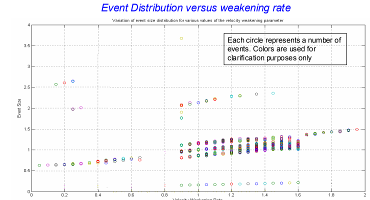
Only hyperbolic -velocity or slip- weakening friction laws led to chaotic behavior for a wide range of weakening rates *(1/v.)*. *Chaos is manifested in the complicated shape of the blocks' phase diagram and in obtaining events of different magnitudes (sizes) for the same weakening rates.*

Orbit in the phase space for one of the blocks for the case of hyperbolic velocity weakening. Trajectories are self intersecting densely as a manifestation of chaos.



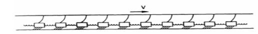
The bifurcation diagram for the event sizes versus the velocity weakening rate interestingly show interchanging regions of periodicity (One characteristic size) and chaos (Different event sizes).

Event Distribution versus weakening rate



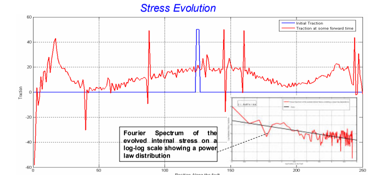
Conclusions:
Hyperbolic friction laws can lead to chaos and complex patterns.
This might be suggestive that some non linear friction laws alone – and regardless the geometric complexity- can lead to stress heterogeneity and scaling laws observed in the earth (which are manifestations of earthquake complexity)

The Several Blocks Model:



The 2-blocks model is too small to uncover the physics of rupture propagation. So, we switched to a system of 250 blocks subjected to a piecewise constant initial traction (the blue line in the figure below) and to hyperbolic velocity dependent friction.

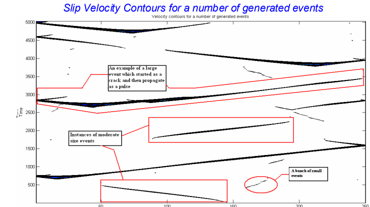
Stress Evolution

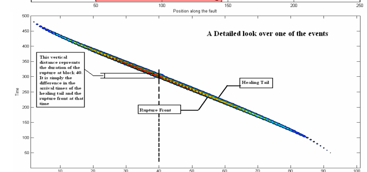


The initial stress evolved into a heterogeneous distribution. This partially supports our intuition about that the stress state in the earth crust should have evolved into a heterogeneous distribution. We expect that if we ran the model for a long enough time and for a larger number of blocks the stress would reach a statistically stable fractal distribution (This is starting to appear in the FT of the evolved internal stress which is showing a power law dependence !).

In order to study how rupture propagates in our model we examined the particle velocity contours for a number of generated events.

Slip Velocity Contours for a number of generated events





This slip duration for a block is generally found to be short compared to the total rupture time for the corresponding event.

Accordingly, the prevailing rupture mode is mainly a propagating pulse.

This might support our intuition that pulses are main ingredients in stress roughening process

Conclusions:
Hyperbolic velocity dependent friction laws lead to events growing mainly as propagating pulses and to an evolving heterogeneity in the stress even if the initial conditions were nearly homogeneous.

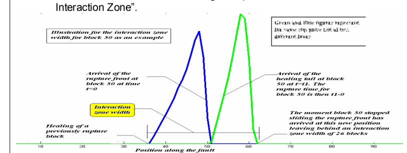
The Proposed Surrogate:

We turn our attention to the study of the observed propagating pulses. Our speculation is that the variation of the pulse width, as the pulse propagates, will lead to a varying stress change resulting in the stress heterogeneity. The pulse would act as an operator which averages the stress over some distance to yield the subsequent slip.

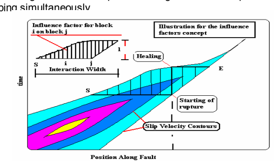
Hence, if we could track the slip pulse width variation and assume an appropriate averaging procedure, we could then predict the slip without solving the equilibrium equations.

To do this we are assuming the following:

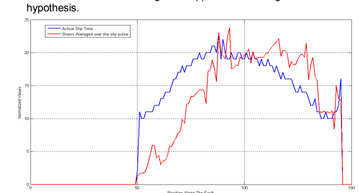
1. Slip duration at any point is affected only by the stresses that lie within a narrow zone surrounding that point which we call "The Interaction Zone".



2. Any point in the interaction zone of a given point affects the slip at that point through a set of influence factors which are proportional to the length of the time period during which both points are slipping simultaneously



3. The actual rupture time at any point in a certain event could be approximated by the value of the stress averaged over the interaction zone of that point using the influence factors as the weighting function.
The results show some good support -on average- for our hypothesis.



width statistically by investigating a promising statistical method known as Markovian Model.

In a Markovian Model, the future state of an observable is predicted from its current state through a probabilistic matrix known as the Transition Matrix.

In our case the observable is the Interaction Zone Width and the Transition Matrix entries at any point are dependent on the sign of the difference in the initial stress between that point and its next one. We are studying the results. Once we are comfortable with our surrogate results, the next step would to carry it over to the continuum models and to extend it to 2D and 3D problems.